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Empirical evidence and stability analysis of the linear car-following model with gamma-distributed memory effect

Xin Pei [a,*], Yan Pan [a], Haixin Wang [b], S.C. Wong [b], Keechoo Choi [c]

[a] Department of Automation, Tsinghua University, Beijing, China
[b] Department of Civil Engineering, The University of Hong Kong, Hong Kong, China
[c] Department of Transportation Engineering, TOD-based Sustainable Urban Transportation, Center, Ajou University, Republic of Korea

HIGHLIGHTS

- A car-following model with stochastic memory effect is considered.
- Empirical evidence is obtained from car-following field experiments.
- Gamma distribution is shown a suitable representation of memory effect empirically.
- General expressions of undamped and stability regions are derived.
- Smaller mean or greater variance of memory effect would enlarge stability region.

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ABSTRACT

Car-following models, which describe the reactions of the driver of a following car to the changes of the leading car, are essential for the development of traffic flow theory. A car-following model with a stochastic memory effect is considered to be more realistic in modeling drivers' behavior. Because a gamma-distributed memory function has been shown to outperform other forms according to empirical data, in this study, we thus focus on a car-following model with a gamma-distributed memory effect; analytical and numerical studies are then conducted for stability analysis. Accordingly, the general expression of undamped and stability points is achieved by analytical study. The numerical results show great agreement with the analytical results: introducing the effect of the driver's memory causes the stable regions to weaken slightly, but the metastable region is obviously enlarged. In addition, a numerical study is performed to further analyze the variation of the stable and unstable regions with respect to the different profiles of gamma distribution.

1. Introduction

Car-following models, which describe the following driver's reaction to the changes of the leading car, are essential for the development of traffic flow theory. Such models build a bridge between the microscopic behavior of the following driver and the macroscopic characteristics of the traffic flow by analysis of the responses of each following vehicle in a single-lane car-following system.

In the 1950s, Chandler et al. [1] and Herman et al. [2] proposed a mathematical car-following model in which it is assumed that the acceleration of the following car in each two-vehicle unit is linearly proportional to the cars' relative velocities at

* Corresponding author. Tel.: +86 10 6279 5043; fax: +86 10 6279 5043.
E-mail addresses: peixin@mail.tsinghua.edu.cn (X. Pei), pany11@mails.tsinghua.edu.cn (Y. Pan), wanghaixin90@126.com (H. Wang), hhecwsc@hku.hk (S.C. Wong), keechoo@ajou.ac.kr (K. Choi).

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some earlier time, with a fixed time lag of transmission of the driver–vehicle system. Herman et al. [2] also provided a systematic discussion of the stability. The results accounted for the local stability and corresponding numerical calculations, and also for the asymptotic stability, car-following control, and acceleration noise. In addition to the linear car-following model, Reuschel [3], Pipes [4], Gazis et al. [5], and Newell [6] developed various nonlinear models in which other variables are included that may affect the behavior of the following driver.

In recent decades, improved car-following models have been proposed to better explain the behavior of drivers and the stability of the traffic flow. Bando et al. [7] proposed an optimal velocity model to represent the instability of traffic flow and the evolution of congestion. Jiang et al. [8] developed a full velocity difference model by differentiation of the deceleration and acceleration processes. Ge et al. [9] extended the car-following model to consider the effects of a series of leading vehicles. In concern of memory effect of time-series variations, the traditional model is developed by incorporating the driver's memory of speeds at or during a certain time ahead [10–12]. Treiber and Helbing [13] concerned memory effect of the subjective level of service dependent on speed, which represents the adaptation of drivers to the surrounding traffic situation during past few minutes, for macroscopic modeling of flow-density data. Yu and Shi [14] adopted empirical data to improve the car-following model by including memory of headway changes, which was further concerned in cooperative adaptive cruise control strategy [15]. Cao [16] also considered the memory effect of headway during a sensory time period during a certain past time. In consideration of the forecast information attributable to ITS techniques, Tang et al. [17] involved driver's forecast effect in car-following model. More recently, with the development of connected vehicles, an improved car-following model with delayed acceleration reaction is proposed [18]. Ngoduy [19,20] and Monteil et al. [21] further focus on the stability of traffic flow in concern of car-following model in a simulated connected vehicle framework.

The time lag, which is related to reaction time, is of great concern in previous studies on macroscopic and microscopic traffic models [22,23]. Generally speaking, the existing car-following models always consider the time lag as a fixed parameter that is estimated in terms of the average or optimal values on the basis of realistic traffic information, even in most memory effect related research mentioned previously. However, in practice, the reactions of the following driver are affected not only by the motion of the vehicles at a certain earlier instant, but also by continuous motions during an earlier period, which can be reflected in the driver's memory, i.e., a memory effect. Classic studies have focused on the memory effects of drivers in the car-following process. Based on the work of Chandler et al. [1] and Herman et al. [2], Lee [24] introduced the function of memory into the traditional car-following model to define the manner in which the following driver processes the information received from the leading car by means of the memory effect. The proposed memory effect model is considered to be more realistic in modeling drivers' behavior.

As shown in Lee's study, the acceleration of the following car is influenced only by the relative speeds. The difference here is that the reactions of the following car are not determined by the relative speeds at a certain earlier instant, but rather by the time history. The memory function is added to the model by the introduction of a stochastic time lag, which may follow a particular distribution over time. Lee [24] described the memory effect with several possible memory functions in his study, such as a Dirac-Delta function (with which the model is reduced to a traditional one), a decaying exponential function (which is also a special case of the gamma-distributed function), another special gamma-distributed function with a concave curve, and a uniform-distributed function. Relevant stability analysis of the local stability and asymptotic stability was also performed in his study. Unfortunately, without experimental data, the advantage of the memory effect model is difficult to assess, although it is considered to be a more realistic means by which to describe the following driver's behavior. In addition, the optimal form of memory function could not be determined. Sipahi et al. [25] have conducted the stability analysis of car-following system with gamma distributed time lag. However, they did not validate the superior performance of gamma distribution, and did not achieve the analytical solution of critical points in local stability analysis.

In this study, we attempt to reveal the most appropriate distribution for memory effect with empirical data. Analytical and numerical studies are then conducted for stability analysis, and the general expression of undamped and stability points is achieved by analytical study, accordingly. A numerical study is performed to further analyze the variation of the stable and unstable regions with respect to the different profiles of memory effect.

The remainder of this paper is organized as follows. In Section 2, after the introduction of the model formulation, several distribution forms of memory effect are proposed. Based on empirical data collected from car-following field experiments, the optimal form of memory effect is obtained. In Section 3, an analytical study of the local stability is conducted to explore the demarcations between stability and instability and between damped oscillation and amplifying oscillation. In Section 4, a numerical study is carried out to show agreement with the analytical results. Section 5 presents the concluding remarks and recommendations for future research.

2. Preliminary study and empirical analysis

2.1. Model formulation

According to the study of Chandler et al. [1], each of the cars in a line of traffic follows the preceding car by means of velocity control. This control equation of motion is

$$M\frac{d^2x_{n+1}}{dt^2} = \lambda\left(\frac{dx_n}{dt} - \frac{dx_{n+1}}{dt}\right)_{t-\Delta}, \tag{2.1}$$