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A small-world and scale-free network generated by Sierpinski Pentagon

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HIGHLIGHTS

- Evolving networks generated by the Sierpinski Pentagon are constructed and analyzed.
- The evolving networks are scale-free.
- The evolving networks have the small-world effect.
- The evolving networks are not fractal scaling.

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ABSTRACT

The Sierpinski Pentagon is used to construct evolving networks, whose nodes are all solid regular pentagons in the construction of the Sierpinski Pentagon up to the stage t and any two nodes are neighbors if and only if the intersection of corresponding pentagons is non-empty and non-singleton. We show that such networks have the small-world and scale-free effects, but are not fractal scaling.

1. Introduction

In the past two decades, lots of attention has been paid to the research of complex networks, which are ubiquitous in nature and society such as WWW [1], metabolic networks [2,3], a network of routers connected by various physical connections [4]. Despite their diversity, most networks appearing in real world obey some organizing principles, which lead to systematic and measurable deviations basing on the random graph theory originating from Erdös and Rényi [5,6]. In particular, two properties of real networks have aroused considerable scientific and technological interests. First, lots of measurements show that most networks display the scale-free property [7], which means that the probability that a randomly selected node with exactly k links decays as a power law, following $P(k) \sim k^{-\alpha}$, where α is the degree exponent. Second, lots of networks have the small-world behavior [8], namely, the geodesic distance between two uniform randomly chosen nodes grows proportionally to the logarithm of the number of nodes, simultaneously they display high average clustering coefficient.

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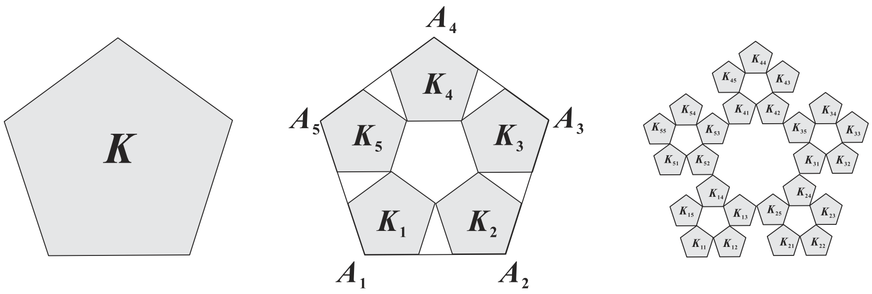

Fig. 1. The first two iterations for the Sierpinski Pentagon.

At the mean time, various self-similar fractals are used to model and mimic evolving complex networks due to the occurrence of self-similarity in real-world complex networks demonstrated by Song et al. [9]. Zhang et al. [10–12] use the Sierpinski gasket to construct evolving networks which follow the power-law degree distribution and possess small-world effect. Several complex networks are modeled on self-similar fractals, for example, Apollonian networks [13], Koch networks [14–18], Platonic solid networks [19], Vicsek networks [20], Sierpinski networks [21,22], the hierarchical networks [23] and generalized self-similar networks [24].

The fractality of complex networks is introduced in Song et al. [9,25], see also Gallos et al. [26]. In a series of articles [27–32], the research group of Makse et al. analyzes the fractality of complex networks and uses their techniques of fractal analysis on networks to study the biological networks. An ℓ_B-box is a subset of node set V such that the distance d_{ij} between any pair of nodes i and j in the box has to be $d_{ij} < \ell_B$. Let $N_B = N_B(\ell_B)$ denote the minimal number of ℓ_B-box to cover V, which is the same as ones in Refs. [9,33]. In Ref. [9], a network with node set V has fractal scaling with fractal dimension d_B if

$$N_B \sim (\ell_B)^{-d_B}. \tag{1.1}$$

In this article, we give a self-similar, scale-free and small-world complex network generated from a self-similar fractal, but our networks are non-fractal scaling, which may be a common feature for some self-similar networks as indicated by Kim et al. [33].

In Section 2, we give out the construction of the complex network generated by Sierpinski Pentagon. Then, in Sections 3 and 4, we show that described network is scale-free and small-world. At last in Section 5, using the technique of fractal geometry, we prove that this network is not fractal scaling.

2. Construction of the network

2.1. Sierpinski Pentagon and its IFS

The Sierpinski Pentagon is generated by the following iteration function system (IFS).

Let $f_i(x) = \frac{3-\sqrt{5}}{2}x + b_i : \mathbb{R}^2 \to \mathbb{R}^2$ for $i = 1, 2, 3, 4, 5$, where $b_1 = \binom{0}{0}$,

$$b_2 = \begin{pmatrix} \dfrac{\sqrt{5}-1}{2} \\ 0 \end{pmatrix}, \qquad b_3 = \begin{pmatrix} \dfrac{\sqrt{5}+1}{4} \\ \dfrac{\sqrt{10-2\sqrt{5}}}{4} \end{pmatrix}, \qquad b_4 = \begin{pmatrix} \dfrac{\sqrt{5}-1}{4} \\ a \end{pmatrix}, \qquad b_5 = \begin{pmatrix} \dfrac{\sqrt{5}-3}{4} \\ \dfrac{\sqrt{10-2\sqrt{5}}}{4} \end{pmatrix},$$

where $a = \frac{2}{3+\sqrt{5}}(2\sin(2\pi/5) + \sin(\pi/5))$, that means the ratio $\frac{3-\sqrt{5}}{2} \approx 0.382$ and

$$b_2 \approx \begin{pmatrix} 0.618 \\ 0 \end{pmatrix}, \qquad b_3 \approx \begin{pmatrix} 0.809 \\ 0.588 \end{pmatrix}, \qquad b_4 \approx \begin{pmatrix} 0.309 \\ 0.951 \end{pmatrix}, \qquad b_5 \approx \begin{pmatrix} -0.191 \\ 0.588 \end{pmatrix}.$$

Then, the Sierpinski Pentagon F is a self-similar fractal satisfying

$$F = \bigcup_{i=1}^{5} f_i(F).$$

Fix the solid pentagon K which is the left one in Fig. 1, then $f_i(K) \subset K$. Let $f_6 = f_1$. Notice that $f_i(K) \cap f_{i+1}(K)$ is a singleton.