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Pricing geometric Asian power options under mixed fractional Brownian motion environment

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HIGHLIGHTS

- Stock price is modeled with driving force as mixed fractional Brownian motion.
- Closed form for the price of the geometric Asian option is derived.
- Closed form for the price of the geometric Asian power option is derived.

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ABSTRACT

It has been observed that the stock price process can be modeled with driving force as a mixed fractional Brownian motion with Hurst index $H > \frac{3}{4}$ whenever long-range dependence is possibly present. We obtain a closed form expression for the price of a geometric Asian option under the mixed fractional Brownian motion environment. We consider also Asian power options when the payoff function is a power function.

1. Introduction

Estimation of option price is an important problem in mathematical finance. A call option is a contract which gives the holder the right but not obligation to buy a risky asset at a certain date called the strike date or the exercise date with a predetermined price called the strike price or the exercise price. A put option is a contract which gives the holder the right but not obligation to sell a risky asset at a certain date called the strike date or the exercise date with a predetermined price called the strike price or exercise price. There are several types of options that are traded in a market. American option allows the owner to exercise his option at any time up to and including the strike date. Bermuda options permit the owner to exercise his option early but only on a contractually specified finite set of dates. European options can be exercised only on the strike date. European options are also called vanilla options. Their payoffs at maturity depend on the spot value of the stock at the time of exercise. There are other options whose values depend on the stock prices over a predetermined time interval. For an Asian option, the payoff is determined by the average value over some predetermined time interval. Asian options reduce the volatility inherent in the option and are cheap compared to the European option (cf. [1,2]). For modeling of fluctuations in movement of stock prices, Brownian motion has been used traditionally as the driving force for modeling log returns. It has been noted later that there might be long-range dependence in the phenomena and the log returns have possibly heavy tailed distributions. It was suggested by some that the driving force for modeling of price

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movement may be chosen as a fractional Brownian motion. Bjork and Hult [3] and Kuznetsov [4] observed that the use of fractional Brownian motion for modeling fluctuations in movement of stock prices is not justifiable as it allows arbitrage opportunities. To avoid this problem, Cheridito [5,6] suggested the use of a mixed fractional Brownian motion as a suitable model to capture the fluctuations of the financial assets. The mixed fractional Brownian motion (mfBm) is a Gaussian process that is linear combination of the Brownian motion and a fractional Brownian motion with Hurst index $H > 1/2$. Cheridito [7] has proved that, for $H \in (3/4, 1)$, the mfBm is equivalent to a Brownian motion and hence modeling price fluctuation via mfBm allows arbitrage-free market. Xiao et al. [8] studied pricing model for equity warrants in a mixed fractional Brownian environment. Sun [9] investigated pricing currency options when the driving force is a mixed fractional Brownian motion. Yu and Yan [10] discussed European call option pricing under a mixed fractional Brownian motion environment. Mao and Liang [1] evaluated geometric Asian option under fractional Brownian motion frame work. They derived a closed form for the solution for the Asian power option price. The pricing of currency options in a mixed fractional Brownian motion in a jump environment has been studied in Refs. [11,12]. Sun and Yan [13] discussed use of mixed-fractional models in credit risk pricing. Our aim is to evaluate the price of Asian power options under a mixed fractional Brownian motion environment.

2. Asian options

The payoff of an Asian option is determined by the average value of the stock price over a pre-fixed time interval as it reduces the risk of market manipulation of the underlying instrument at maturity and reduce the volatility in the option. Furthermore, Asian options are generally cheaper than the corresponding European options. Asian options are of different types such as fixed strike price options and floating strike price options. The payoff for a fixed strike price option is $(A(T) - K)_+$ and $(K - A(T))_+$ for a call and put option respectively where K denotes the strike price, T is the strike time and $A(T)$ is the average price of the underlying asset over the predetermined interval. For a floating strike price option, the payoffs are $(S(T) - A(T))_+$ and $(A(T) - S(T))_+$, for a call and put option respectively where $S(T)$ is the price of stock at time T. Asian options can again be differentiated in to two classes: one is the arithmetic average, that is,

$$A(T) = \frac{1}{T}\int_0^T S(t)\mathrm{d}t$$

and the other is the geometric average

$$A(T) = \exp\left\{\frac{1}{T}\int_0^T \log S(t)\mathrm{d}t\right\}$$

assuming that the pre-fixed interval for computing the average is the interval $[0, T]$. We will consider evaluation of Asian option price in the case of continuous geometric average with a fixed strike price in an mfBm environment.

3. Mixed fractional Brownian motion

We now define the mixed fractional Brownian motion (mfBm) and discuss some of its properties.

A mixed fractional Brownian motion $M^H(\alpha, \beta)$ is a linear combination of a Brownian motion and a fractional Brownian motion (fBM) with Hurst index H, that is,

$$M_t^H(\alpha, \beta) = \alpha W_t + \beta W_t^H, \quad 0 \le t < \infty \tag{3.1}$$

where W is the standard Brownian motion and W^H is an independent standard fractional Brownian motion with Hurst index H and α, β are some real constants not both zero. The equality here is understood in the sense that the finite dimensional distributions of the process on the left side of Eq. (3.1) are the same as the corresponding finite dimensional distributions of the process on the right side of Eq. (3.1). The process $M^H(\alpha, \beta)$ is a centered Gaussian process with $M_0^H = 0$ a.s. and with the covariance function

$$cov(M_t^H, M_s^H) = \alpha^2 \min(t, s) + \frac{\beta^2}{2}(t^{2H} + s^{2H} - |t - s|^{2H}).$$

The increments of the process $M^H(\alpha, \beta)$ are stationary and self-similar, in the sense that, for any $h > 0$,

$$M_{ht}(\alpha, \beta) \stackrel{\Delta}{=} M_t^H(\alpha h^{1/2}, \beta h^H). \tag{3.2}$$

Here Δ indicates that the random variables on both sides of Eq. (3.2) have the same distribution. The increments of the process are positively correlated if $\frac{1}{2} < H < 1$, uncorrelated if $H = \frac{1}{2}$ and negatively correlated if $0 < H < \frac{1}{2}$. The increments of the process are long-range dependent if and only if $\frac{1}{2} < H < 1$. For more details on the properties of a mfBm, see Refs. [14,15]. We assume here after that the index $H > \frac{3}{4}$ which ensures that the probability measure generated by the process $M^H(\alpha, \beta)$ is equivalent to the Wiener measure. For simplicity in computations, we assume that $\alpha = \beta = 1$ here after. Integration of an adapted process with respect to the mixed fractional Brownian motion is defined as the sum