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Non-equilibrium thermo-field dynamics for anti-ferromagnetic spin system

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HIGHLIGHTS

- NETFD for an anti-ferromagnetic system coupled with a phonon reservoir is proposed.
- The two kinds of quasi-particle operators are introduced and their forms are derived.
- Forms of the spin correlation functions and longitudinal magnetization are derived.
- The energy, magnetization and correlation functions are investigated numerically.
- The two-point Green's function is derived in a form of 4×4 matrix.

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ABSTRACT

The non-equilibrium thermo-field dynamics for an anti-ferromagnetic spin system interacting with a phonon reservoir is proposed for the case of a non-bilinear unperturbed Hamiltonian, which includes not only a bilinear part but also a non-bilinear part, in the spin–wave approximation. The two kinds of quasi-particle operators are introduced, and their forms are derived for the semi-free boson fields. It is shown that the two quasi-particles decay exponentially with the frequencies and life-times which are different from each other. It is also shown that each quasi-particle changes to the other tilde quasi-particle through the spin–phonon interaction. The spin–spin correlation functions and longitudinal magnetization for the anti-ferromagnetic spin system under an external static magnetic field are derived in the forms convenient for the perturbation expansions. The expectation values of the spin–wave energy and longitudinal magnetization and the spin–spin correlation functions are investigated numerically for an anti-ferromagnetic system of one-dimensional infinite spins interacting with a damped phonon-reservoir, in the region valid for the lowest spin–wave approximation and the narrowing-limit approximation in which the relaxation times of the spin system are much larger than the correlation time of the phonon reservoir. The two-point Green's function of the semi-free spin–wave for the anti-ferromagnetic spin system is derived by introducing the thermal quartet notation, and it is given in a form of 4×4 matrix.

1. Introduction

The equilibrium thermo-field dynamics [1–7] formulated by extending quantum field theory to the case of finite temperature has many useful properties of quantum field theory, e.g., the operator formalism, the time-ordered formulation

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of the Green functions, the Feynman diagram method in real time, etc., and is useful to treat many-body systems, because the statistical average is given by the expectation value in the thermal vacuum state. Arimitsu and Umezawa proposed the non-equilibrium thermo-field dynamics (NETFD) by combining the concepts of the coarse-graining with the thermal state and by introducing the thermal-Liouville space, for a quantum system interacting with its heat reservoir [8–10]. Arimitsu and Umezawa formulated the NETFD for the case of a bilinear unperturbed Hamiltonian of the system and reservoir [8–10].

When the interaction of a boson system with its heat reservoir includes not only a non-adiabatic part but also an adiabatic part [11], the unperturbed Hamiltonian for the boson system and heat reservoir includes not only a bilinear part but also a non-bilinear part. In Ref. [12], one of the authors (MS) generalized the non-equilibrium thermo-field dynamics to the case of a non-bilinear unperturbed Hamiltonian which includes not only a bilinear part but also a non-bilinear part, and applied it to a ferromagnetic spin system [13]. He formulated the non-equilibrium thermo-field dynamics for a ferromagnetic spin system in the spin–wave approximation [14] in Refs. [13,15], studied the temperature dependence and wave number dependence of line shape of the transverse susceptibility for the ferromagnetic spin system, and found some interesting phenomena [13,15]. In particular, in Ref. [13], he found that the non-bilinear part of the unperturbed Hamiltonian for the spin system and reservoir gives the temperature dependence of the line shape. In Refs. [13,15], he studied the transverse susceptibility for the ferromagnetic spin system by using the TCLE method proposed by himself [16–23]. The TCLE method, in which the admittance of the system is directly calculated from time-convolutionless (TCL) equations including the terms which come from an external driving field, is a convenient formalism to calculate the complex admittances by using the non-equilibrium thermo-field dynamics [12,13,15,21–24]. The reason is as follows. The non-equilibrium thermo-field dynamics [8–10,12] has been formulated assuming the van Hove limit approximation [25] or the narrowing limit approximation [26] for the system–reservoir interaction, in which the effects of the initial correlation and memory for the system and reservoir are neglected. The effects of the initial correlation and memory produce effects that cannot be ignored in general [15,19,20,27–29]. In the TCLE method [16–24], the admittance of the system includes the effects of the initial correlation and memory for the system and reservoir, which are the effects of the deviation from the van Hove limit [25] or the narrowing limit [26] and are represented by the interference terms or the interference thermal states in the TCL equations with external driving terms. When the TCLE method is used, the admittance of the system can be calculated by inserting the corresponding interference terms into the results obtained in the van Hove limit [25] or in the narrowing limit [26]. Thus, the admittance of the system can be calculated by using the non-equilibrium thermo-field dynamics and the TCLE method. It may be an interesting problem to study non-equilibrium properties not only for a ferromagnetic spin system but also for an anti-ferromagnetic spin system. Therefore, it may be necessary to formulate the non-equilibrium thermo-field dynamics for an anti-ferromagnetic spin system.

The resonance absorption of the anti-ferromagnetic spin systems was macroscopically treated by Nagamiya [30], Kittel and Keffer [31], and was microscopically discussed using the spin–wave method [14] by Nakamura [32], Ziman [33], Kubo [34], Akhiezer et al. [35] and Oguchi and Honma [36]. The anti-ferromagnetic resonance was also discussed using the method of the collective motion of spins by Mori and Kawasaki [37], and was studied numerically using the method of calculating the dynamical susceptibility directly by Miyashita et al. [38], and its theories were developed by the quantum field theoretical approach of Oshikawa and Affleck [39] to the electron spin resonance in spin-1/2 chains. However, these theories for anti-ferromagnetic resonance absorption do not deal with the effects of the phonon reservoir interacting with the spin systems, and therefore those theories cannot elucidate the damping mechanism of the spin for the case that the spin–spin interactions or the spin–wave interactions are small. In such a case, it is necessary to consider the anti-ferromagnetic spin systems interacting with the phonon reservoirs and to study the effects of the phonon reservoir. Uchiyama et al. [40] proposed a method in which the Kubo formula [41] is calculated using the time-convolution (TC) master equation to study effects of the heat reservoir, and applied it to a two-spin system and a three-spin system. If the non-equilibrium thermo-field dynamics is formulated for anti-ferromagnetic systems of many spins, it may become a good tool.

In the present paper, we consider an anti-ferromagnetic spin system with a uniaxial anisotropy energy and an anisotropic exchange interaction under an external static magnetic field in the spin–wave region, interacting with a phonon reservoir. The interaction between the spin–wave and phonon is assumed to include not only a bilinear part but also a non-bilinear part, which corresponds to the interaction between the z components of the spin and phonon, in the lowest spin–wave approximation. We formulate the non-equilibrium thermo-field dynamics by introducing the two kinds of quasi-particle operators for such an anti-ferromagnetic spin–wave system, and derive forms of the quasi-particle operators for the semi-free boson fields. We also derive the spin–spin correlation functions and longitudinal magnetization for the anti-ferromagnetic spin system under an external static magnetic field, in the forms convenient for the perturbation expansions. We besides investigate numerically the expectation values of the spin–wave energy and longitudinal magnetization and the spin–spin correlation functions for an anti-ferromagnetic system of one-dimensional infinite spins, in the lowest spin–wave approximation. Moreover, we derive a form of the two-point Green's function of the semi-free spin–wave for the anti-ferromagnetic spin system.

We here define the notations of thermo-field dynamics. In the Liouville space and thermal Liouville space [8–10,42,43] of a quantum system and its quantum heat reservoir, the tilde conjugate $\tilde{A}$ of an operator A is defined to satisfy the following rules [8–10,44]:

$$(AB)^{\sim} = \tilde{A}\tilde{B}, \qquad (c_1A + c_2B)^{\sim} = c_1^*\tilde{A} + c_2^*\tilde{B}, \qquad \tilde{A}^{\dagger} = (A^{\dagger})^{\sim}, \qquad (\tilde{A})^{\sim} = A, \tag{1.1}$$

for arbitrary operators A and B and for arbitrary complex c-numbers c_1 and c_2.