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An optimal control model for reducing and trading of carbon emissions

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ABSTRACT

A stochastic optimal control model of reducing and trading for carbon emissions is established in this paper. With considerations of reducing the carbon emission growth and the price of the allowances in the market, an optimal policy is searched to have the minimum total costs to achieve the agreement of emission reduction targets. The model turns to a two-dimension HJB equation problem. By the methods of reducing dimension and Cole–Hopf transformation, a semi-closed form solution of the corresponding HJB problem under some assumptions is obtained. For more general cases, the numerical calculations, analysis and comparisons are presented.

1. Introduction

In recent years, global warming, climate change and carbon emission management have been a universal concern. In 1997, the Kyoto protocol was signed at the conference in Kyoto, and assigns mandatory emission limits for greenhouse gases, primarily carbon dioxide (CO_2) to the signatory nations [1]. To implement the Kyoto Protocol in European Union, the European Commission launched European Climate Change Program (ECCP) in Jun. 2000. In Jan. 2005, the European Union Emission Trading Scheme (EU ETS) [2] began operating. This is the first and still by far the biggest international system for trading greenhouse gas emission allowances and covers around 45% of total greenhouse gas emissions from the 28 EU countries.

The EU ETS works on the cap-and-trade principle. A cap means that the total amount of greenhouse gas can be emitted by the country, or a factory specifically in a predetermined period. If the signatory parties run out the cap, they need to purchase the EUA (the European Union Allowance) from the others. They can also sell the spare allowances in the market to make profits [2,3]. Furthermore, it allows the signatory parties to buy the international credits from emission-saving projects that carried out under CDM and JI (The Kyoto Protocols Clean Development Mechanism and Joint Implementation) instrument around the world.

Nowadays, several national and regional emission markets have been established and have a remarkable growth. Meanwhile, there have been a lot of papers proposing the research in different aspects. Daskalakis et al. [4] studied the three main markets for emission allowances within the EU ETS. They develop a framework for the pricing and hedging of futures and options on futures. Carmona et al. [5] showed that the economic mechanism of carbon allowance price formation can be formulated in the framework of competitive stochastic equilibrium models and identify the main allowance price drivers. Wang et al. [6] analyzed enterprises' emission reduction pathways in practical process based on China's emission abatement target, and develops an framework to derive the magnitude of investment required in each pathway. Zagheni and

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Billari [7] used a so called STIRPAT model to describe the trends in carbon dioxide emissions on the basis of economic and demographic dynamics. They also built a model in this framework to evaluate the costs of a country reducing the emission to reach the cap from the aspect of option pricing. Yang and Liang [8,9] proposed an optimal control model. They used a stochastic process to describe the carbon emission of a country and set the objective to minimize the total costs, which included the costs in reducing emission and the penalties of running out the cap according the EU ETS. They solved the problem by Hamilton–Jacobi–Bellman (HJB) equations. The existence and uniqueness of the classical solution were also proved in Ref. [9].

In this paper, we consider the optimal control problem with the carbon emission trading in the Merton's optimal consumption framework [10,11]. Describing the carbon emitting process by a modified STIRPAT model [9], the country needs to meet emissions target by reducing the carbon emissions, and trading the allowances in the termination time which means buying the allowances if the emission quota is exceeded and selling the allowances if not. The goal of the decision maker is to get an optimal reducing policy with minimum total costs and, accordingly, a stochastic optimal control model is established. By the methods of reducing dimension and Cole–Hopf transformation, we solve the corresponding HJB problem and obtain a semi-closed form solution under some further assumptions. For general case, a numerical result is presented. It shows that, for small perturbation, the semi-closed form solution can be approximated to the original solution.

The remainder of the paper is organized as follows: In Section 2, we give model assumptions and establish an optimal control model to minimize the total costs while considering the carbon emission abatement and trading. Meanwhile, a semi-closed form solution of the corresponding HJB problem is given. In Section 3, numerical calculation results and analysis are shown for more general cases. Section 4 concludes the paper.

2. The optimal control model

To describe the trend of carbon emission of a country, we adopt the modified STIRPAT model [8] which can be traced back to Ref. [12] and it is an improved model of classical IPAT model [13]. In the modified STIRPAT model, the carbon emission of an area is determined by the economy and the population. Here, we use the GDP, denoted by Y_t, representing the economy level of the country, and assume it satisfies Geometric Brownian Motion (GBM):

$$dY_t = Y_t\mu_1 dt + Y_t\sigma_1 dW_t^1 \tag{1}$$

where μ_1 is the drift rate, σ_1 is the volatility and they are both positive constants, W_t^1 is the standard Brownian motion. Similar assumption can also be found in Refs. [7,14], etc.

We use the Logistic model as Yang and Liang [9] to describe the population. Assume that the population of a country at time t is P_t, the carrying capacity of the population of the country is P_m and the intrinsic population growth is $\hat{\rho}$. Thus, the actual population growth rate should be

$$\begin{cases} \dfrac{P'}{P} = \hat{\rho}\left(1 - \dfrac{P}{P_m}\right), \\ P(0) = P_0. \end{cases}$$

By solving the ordinary differential equation standardly, we have

$$\frac{dP_t}{P_t} = \frac{\hat{\rho}(P_m - P_0)}{P_0 e^{\hat{\rho}t} + P_m - P_0} dt := f(t)dt. \tag{2}$$

Let I_t be the carbon emission of a country in one year. Suppose the initial emission amount $I_0 > 0$. In the modified STIRPAT model, the process of carbon emission can be expressed as

$$\frac{dI_t}{I_t} = a_1\frac{dY_t}{Y_t} + a_2\frac{dP_t}{P_t} \tag{3}$$

where a_1 and a_2 are the constant parameters, representing the contributions by the population and GDP respectively. From the expressions of the growth rate (2) and the growth rate of GDP (1), the Eq. (3) can be rewritten as

$$dI = I(a_1\mu_1 + a_2 f(t) - q_t)dt + Ia_1\sigma_1 dW_t^1$$

where $q_t > 0$ is the control policy, represents the reduced growth rate of carbon emission. We define that the admissible controls $\mathcal{Q}$ is the set of all q which satisfies $\int_t^T g(q_s)ds < \infty$.

As in EU ETS, the countries can trade the emission allowances with each other. In this paper, we assume the price process C_t of emission allowances in the emission trading market satisfies the GBM:

$$dC = C\mu_2 dt + C\sigma_2 dW_t^2$$

where μ_2 is the constant drift parameter and σ_2 is the constant volatility parameter. W_t^2 is the Brownian motion and assume that

$$dW_t^1 dW_t^2 = \rho dt.$$