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Diffusion inspires selection of pinning nodes in pinning control

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HIGHLIGHTS

- Large-degree pinning nodes perform badly in real-world networks.
- Pinning node selection is transferred into multi-spreaders in information diffusion.
- Poor performance of large-degree selection is due to overlapping influence of pinning nodes.
- An effective pinning node selection is proposed to enhance pinning controllability.

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ABSTRACT

The outstanding problem of controlling a complex network via pinning is related to network dynamics and has the potential to master large-scale real-world systems as well. This paper addresses the heart issue about how to choose pinning nodes for pinning control, where pinning control aims to control a network to an identical state by injecting feedback control signals to a small fraction of nodes. We explore networks' controllability from not only mathematical analysis, but also the aspects of network topology and information diffusion. Then, the connection between pinning control and information diffusion is given, and pinning node selection is transferred into multi-spreader problem in information diffusion. Based on information diffusion, a heuristic method is proposed to select pinning nodes by optimizing the spreading ability of multiple spreaders. The proposed method greatly improves the controllability of large practical networks, and provides a new perspective to investigate pinning node selection.

1. Introduction

Many natural, social, and man-made systems can be considered as complex networks, of which nodes represent elements of the systems and edges indicate relations between elements [1–3]. Since computer science and Internet provide an easy access to large network datasets, complex network attracts scientists from diverse disciplines (e.g. computer science, physics, biophysics, and social sciences), and much work has been devoted to studying the relationship between topology

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and dynamics including synchronization, diffusion of rumors, virus propagation, spreading of neural signals, *etc.* [4–7]. Among these, controlling the dynamics is ultimate proof of understanding networks, in which a key issue is that how to reduce the cost of controllability.

In nature, collective behaviors (e.g. flocking, swarming and schooling) emerge in groups such as ants, birds, fish, bees and so on [8], in which a small fraction of agents could guide the whole nodes to a desired state. These natural phenomena inspire that we can drive a network from any arbitrary initial state to a desired state by only controlling a small set of nodes [8,9]. The set of controlled nodes could be determined by roles or attributes (e.g. age, sex, interests and so on) of the units in practical natural networks. But for a network with only edge connections *prior*-known, how to select the controlled nodes is still a challenge and it attracts many scientists to work on it [8,10–12]. Among those, Wang et al. studied pinning control of networks and the results showed that controlling high degree nodes was better than that of random selection in scale-free artificial networks [9,13]. Jalili et al. explored the optimal pinning nodes, which was only appropriate for small network due to calculation complexity [14]. Apart from pinning control (non-linear system), many researchers aim to control each node to any arbitrary final state in linear systems (full control for short): Liu et al. developed an analytical tool to select driver nodes and found that the minimum number of driver nodes required to control networks was determined by the degree distribution [10,15]. Yuan et al. explore the exact controllability in complex networks and multiplex networks [16,17]. Wang et al. perturbed network structure to optimize its controllability [18]. Tang et al. identified controlled nodes in neuronal networks and found a transition in choosing driver nodes from large degree to low degree [19]. Since pinning control is a particular case of full control, some achievement in full control may be also capable for pinning control.

Unlike previous work, in this paper, we firstly find an abnormal phenomenon that large-degree selection method performs worse than random selection in real-world networks. Then, the relationship between pinning control and information diffusion is built, by which the problem of pinning node selection is transferred into multi-spreaders in information diffusion that has been investigated previously [20–22]. Based on information diffusion, a novel and simple method is proposed to select pinning nodes. Our method selects pinning nodes according to the overall influences of pinning nodes. Note that, the overall influences are not the sum of each node's influence due to the overlapping influence between nodes. The proposed method enhances the controllability a lot compared to that of traditional large-degree selection. What is more, our method provides a new perspective (information diffusion) to study pinning nodes selection and could inspire many other effective heuristic algorithms in the future.

This paper is organized as follows: To better motivate the problem, in Section 2 we begin with the theory of pinning control and find the poor controllability of conventional large-degree selection in practical networks. Then in Section 3 we introduce the connection between pinning control and information diffusion. Later, in Section 4 we give the reason why large-degree selection method achieves bad controllability in real-world networks from the perspective of information diffusion. Next, in Section 5 a novel method is proposed to select pinning nodes, which enhances the controllability of real-world networks remarkably. The conclusion is given at Section 7.

2. Pinning control

To be able to analyze the dynamics of a system, the state equations of a network containing N identical linearly and diffusively coupled nodes can be written as [9]:

$$\dot{\mathbf{x}}_i = f(\mathbf{x}_i) + c\sum_{j=1}^{N} a_{ij}\Gamma\mathbf{x}_j, \quad i = 1, 2, \ldots, N, \tag{1}$$

where $\mathbf{x}_i = (x_{i1}, x_{i2}, \ldots, x_{in})'$ are the state variable of node i, c is the coupling strength ($c > 0$), a_{ij} represents the elements of the adjacent matrix $\mathbf{A}$, and $\Gamma \in R^{n\times n}$ is a diagonal matrix ($\Gamma = \mathrm{diag}(r_1, r_2, \ldots, r_n)$, $\Gamma \geq 0$) linking coupled variables. Note that $a_{ij} = a_{ji} = 1$ if there is an edge between node i and j ($i \neq j$), otherwise $a_{ij} = a_{ji} = 0$. The element a_{ii} on the diagonal is $a_{ii} = -k_i$, where k_i is the degree of node i.

The aim is controlling the network toward a homogeneous stationary state ($\mathbf{x}_1 = \mathbf{x}_2 = \cdots = \mathbf{x}_n = \bar{\mathbf{x}}$) by applying linear feedback injections to a small fraction δ of nodes. Suppose $i_1, i_2, \ldots, i_l$ ($l = \lfloor N\delta \rfloor$) nodes are controlled, the state equations are modified as [9]:

$$\dot{\mathbf{x}}_{i_k} = f(\mathbf{x}_{i_k}) + c\sum_{j=1}^{N} a_{i_k j}\Gamma\mathbf{x}_j - cd\Gamma(\mathbf{x}_{i_k} - \bar{\mathbf{x}}), \quad k = 1, \ldots, l, \tag{2a}$$

$$\dot{\mathbf{x}}_{i_k} = f(\mathbf{x}_{i_k}) + c\sum_{j=1}^{N} a_{i_k j}\Gamma\mathbf{x}_j, \quad k = l+1, l+2, \ldots, N, \tag{2b}$$

where $d > 0$ is the feedback gain. The system can stabilize into the stationary state $\bar{\mathbf{x}}$ if [9,8]

$$c \geq \left|\frac{\rho}{\lambda_1}\right|, \tag{3}$$