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Controlling the non-classical properties of a hybrid Cooper pair box system and an intensity dependent nanomechanical resonator

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HIGHLIGHTS

- The work treats a hybrid system (CPB–NR) via the Buck–Sukumar model.
- Quantum entropy and excitation–inversion are focused.
- System evolution in presence of losses is considered.

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ABSTRACT

The present work refers to a somewhat realistic treatment to investigate the entropy and the excitation–inversion of a coupled system that consists of a nanomechanical resonator and a superconducting Cooper pair box. The procedure uses the Buck–Sukumar model in the microwave domain and considers the nanoresonator with a time-dependent frequency as well as both subsystems in the presence of losses. The results were obtained for the temporal evolutions of the entropy of each subsystem and the excitation–inversion of the Cooper pair box. A comparison about which of these two subsystems is more sensitive to the presence of losses was done. The results suggest that appropriate choices of the time-dependent parameters can help us to monitor these properties of the subsystems and may offer potential applications, e.g., in the generation of non-classical states, quantum communication, and quantum lithography.

1. Introduction

In the recent years investigations on nanomechanical systems [1] have rapidly been developed. The rush in this direction was stimulated by various perspectives and applications, which before were unsuspected. The enormous progress in the research of nanomechanical systems has also made the dream of controlling the interface between the quantum and classical worlds in a realistic way. This was shown by the emergence of hybrid quantum systems [2], which are intended to achieve a coherent transfer of quantum information from a single quantum emitter (e.g., superconducting qubits, cooper pair box,

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microwave resonators, quantum dots, etc.) and a solid state mechanical resonator, which moves to quantum physics giving birth to a new paradigm. In addition, these systems have received considerable attention due to their diverse potential applications, including metrology (mass force or spin ultra-sensitive detectors), based on the remarkable sensing properties of nanomechanical resonators and their very low mass and lightness.

An important focus of quantum optics is concerned with the atom–field system. Inspired by the various tests applied to this coupled system and on the several results obtained, including their limitations, the researchers have passed from the light domain to the microwave domain of the superconducting version, the quantum electro-dynamics circuit. This system furnishes a new test for microwave domain that interacts with superconducting qubits[3,4]. For a broader context, we can mention investigations using this system, e.g., Landau–Zener transition [5]; atomic physics and quantum optics [6]; mechanisms for photon generation from quantum vacuum [7]; quantum simulation, challenges, and promises of fast-growing field [8].

Here we assume that in the laboratory the atom is substituted by the Cooper pair box (CPB) and the photon is substituted by the nanomechanical resonator (NR) [9]. So, the atom–field interacting system goes to the **CPB–NR** interacting system with concomitant passage from the optical domain to the microwave domain. Cooper pair boxes coupled to nanoresonators have been analyzed in the following works: quantum network [10]; squeezed states and entangled states [11]; cooling mechanical oscillators [12]; Bell inequality violations [13].

There are few works in supporting literature that deal with the interaction between a CPB and an NR when the latter has either a time-dependent frequency [14,15] or a time-dependent amplitude of oscillation. The well-known Jaynes–Cummings model (*JCM*), which describes the interaction of a single two-level atom and a single mode of a quantized radiation field, is the simplest model for this system and provides exact solutions. It is analogous to the **CPB–NR** system, but since the year 1963, many others studies have implemented the *JCM* to describe the atom–field interaction [16]. Some generalized models were also constructed and extensively studied [17–19]. These studies include the system acted upon by the Stark effect [20,21] to investigate quantum non-demolition measurements [22–24]. Usually the investigations assume the field initially in a (pure) coherent state. However, as the atom–field interaction is turned on the field state changes with the time evolution; while the field state loses its coherence it becomes non-classical [25]. Some references to this subject are, e.g., about non-classical properties of a state [26], generation of superposition states [27], the degree of non-classicality of a state [28] and: sculpturing coherent states to get Fock states [29] — plus references therein.

The traditional *JCM* was extended to the case of intensity-dependent coupling, proposed by B. Buck and C.V. Sukumar [30] in order to study the influence of the field intensity, via its excitation number, upon the atom–field system, where a single atom interacts with a single mode of an optical field. This model was generalized by V. Buzek [31] to include a new coupling, with time-dependent intensity. In the present work we will employ the model by Buck–Sukumar (*BS*) to study the **CPB–NR** system, namely: we will suppose the coupling being dependent of the intensity of the **NR** oscillations and also that it changes with time, as assumed in Ref. [31]. In addition, we include the presence of dissipation effects to put the system in a realistic scenario and verify in which way they affect the excited level of the **CPB**, the excitations of the **NR**, and the dynamical properties of the entire **CPB–NR** system; then some points raised are: how dissipation spoils the system operation and in which way the detuning could prevent it, allowing us the control of entanglement features, collapse–revival effects, and others. The results obtained indicate the possibility of some potential applications [32–37].

2. The Hamiltonian system

A superconductor **CPB** charge qubit is adjusted to the input voltage V_1 of the system, through a capacitor with an input capacitance C_1. Following the configuration shown in Fig. 1 we observe three loops: a small loop in the left, another in the right, and a great loop in the center. The control of the external parameters of the system can be implemented via the input voltage V_1 and the three external fluxes Φ_L, Φ_r and Φ_t. The control of these parameters allows us to make the coupling between the **CPB** and the **NR**. We consider $\hbar = 1$ and assume as identical the four Josephson junctions of the circuit system, having the same Josephson energy E_J^0; the external fluxes Φ_L and Φ_r are also assumed as identical in magnitude, although they have opposite signs $\Phi_L = -\Phi_r = \Phi_x$ (see Ref. [14]). So, taking into account the decay in the excited level of the **CPB** and dissipation in the **NR**, we can write the total Hamiltonian of the system as follows,

$$\hat{H} = \omega(t)\hat{a}^\dagger\hat{a} + \frac{1}{2}\omega_c(t)\hat{\sigma}_z + \lambda(t)\left(\hat{a}\sqrt{\hat{a}^\dagger\hat{a}}\hat{\sigma}_+ + \sqrt{\hat{a}^\dagger\hat{a}}\hat{a}^\dagger\hat{\sigma}_-\right) - i\gamma(t)|e\rangle\langle e| - i\delta(t)\hat{a}^\dagger\hat{a}. \quad (1)$$

In Eq. (1) the first term describes the **NR**, the second describes the **CPB**, the third represents the intensity dependent interaction introduced by the *BS* model, here including the time-dependent parameter $\lambda(t)$ introduced by Buzek, the fourth term $\gamma(t)$ stands for the time-dependent loss affecting the **CPB** and the fifth term $\delta(t)$ stands for the time-dependent loss that affects the **NR**. In this equation $\hat{a}^\dagger$ ($\hat{a}$) is the creation (annihilation) operator for excitations in the **NR**, $\hat{\sigma}_+$ ($\hat{\sigma}_-$) is the raising (lowering) operator for the **CPB**, and $\hat{\sigma}_z$ is the z-component of the Pauli spin operator. Eq. (1) stands for a non-Hermitian Hamiltonian (*NHH*). Usually, quantum mechanics works with Hermitian Hamiltonians; however, one can find many papers in supporting literature using *NHH*: one of them appears in Ref. [38], used by the authors to map a wave guide system, metal-silicon, where an optical potential is modulated along the length of the wave guide to get a non-reciprocal light propagation; another application appears in Ref. [39], employed by the authors to show occurrence of entanglement