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Quantum entanglement and quantum phase transition for the Ising model on a two-dimension square lattice

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HIGHLIGHTS

- The zero-temperature quantum entanglement around the quantum critical point for the Ising model on a square lattice is investigated.
- The quantum entanglement exhibits some interesting behaviors such as nonanalytic and scaling behaviors.
- The scaling relationship between the entanglement exponent and the correlation length exponent is also found.

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ABSTRACT

The quantum entanglement and quantum phase transition of the transverse-field Ising model on a two-dimensional square lattice were investigated by applying the quantum renormalization group method. The quantum critical point (QCP) and the correlation length exponent, ν, were obtained. By taking the concurrence as a measure of entanglement, the entanglement between spin blocks near the QCP is calculated as the size of the system becomes large. The entanglement reaches a maximum close to QCP, and can exist in a small range around QCP just at the limit of thermodynamics. The nonanalytic behavior of the derivative of the entanglement with the external field shows that the system undergoes a second order quantum phase transition from a ferromagnetic phase to a paramagnetic phase. The finite-size scaling behavior of the entanglement is described, and the relationship between the entanglement exponent, θ, the correlation length exponent, ν, and the dimension of the system d is also found, i.e., $\theta = 1/(\nu d)$.

1. Introduction

Quantum entanglement, which has been regarded as an important concept since the early age of quantum mechanics (1930s) [1–4], has been exploited as a crucial resource to realize quantum communication and quantum computation over the past two decades [5–7]. Because of its ability to capture the nonlocal correlation in many-body systems, the entanglement has been extensively investigated in other fields, for example, condensed matter physics and even biophysics [8–11]. These related studies will be very useful in understanding the mechanism for entanglement and to apply entanglement theory to quantum information processing. Recently, the relationship between entanglement and the quantum phase transition (QPT) in many-body systems has been widely investigated [12–14]. The QPT takes place at zero temperature as some external variables or the coupling constants change [15]. The QPT is induced by quantum fluctuation which is driven by the Heisenberg uncertainty principle. The correlation length diverges close to the quantum critical

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point (QCP), which reveals that the different parts of the quantum system are strongly correlated. Typical low-temperature examples of QPT in real systems include the transitions in superconductors, quantum Hall systems, and Bose–Einstein condensates. As novel low-dimensional nano-materials are continually prepared in experiments, the amazing properties of these systems especially in the vicinity of the QCP have stimulated a good number of theoretical and experimental studies [16]. The entanglement has been considered a new and effective tool to characterize the quantum critical behaviors in this emerging field.

Solid-state spin systems can describe various real materials which display rich critical behaviors, and the entanglement properties related to QPT in these systems have been extensively studied [14]. Initial reports by Osborne and Nielsen [12], and Osterloh et al. [13] found the singular and scaling behaviors of pairwise entanglement close to the QCP of the Ising spin chain. Based on these results, the pairwise or block entanglements have been applied to study the QPT in one-dimension (1D) systems by analytic or numerical methods [17–23].

Advancing beyond the 1D case, some further contributions to the entanglement on two-dimension (2D) spin systems have been recently made. Some numerical methods, such as Monte Carlo simulation, trace minimization, and tensor renormalization group method, have been developed to examine the entanglement in 2D Ising, XY, and XXZ models [14,24–36]. While the cusp maximum of entanglement around the QCP on the 2D spin systems was found, the scaling behavior of entanglement and the relationship between the critical entanglement and the correlation length need to be further studied.

In the past few years, the renormalization group (RG) methods have been successfully used to study the quantum correlation properties in spin systems especially as the QPT occurs. In 1D spin chains at zero temperature, the quantum renormalization group (QRG) method provides new insights into how the block entanglement changes as the size of the system becomes large. The nonanalytic behavior of entanglement and its scaling behaviors are also naturally revealed by this method [17–21]. This QRG method has been generalized to analyze the spin models on higher-dimensional lattices including square, triangular, and 3D cubic lattices [37,38]. It was found that this analytical method can capture long-distance correlation properties, and obtain the accurate estimation of the critical exponent for the correlation length.

Motivated by this QRG approach, we studied the entanglement and QPT in a 2D transverse-field Ising model. It was found that there exists a definite relationship between the scaling behavior of the entanglement and the divergence of correlation length close to the critical point. This paper is organized as follows. In Section 2 we first review how to employ QRG method to deal with the Ising model on a square lattice. The properties of block–block entanglement, its nonanalytic and scaling behaviors are examined in Section 3. The summary is given in Section 4.

2. Quantum renormalization group for the square lattice

We first briefly introduce the QRG method, and its key idea, which is to keep the most important degrees of freedom in the low-energy spectrum while eliminating the rest through an iterative process. For this reason, the original Hamiltonian H can be mapped onto a renormalized Hamiltonian H_{eff}, which is defined by a set of renormalized coupling constants. The QRG method used in this paper is implemented by Kadanoff's block-site transformation. Generally, there are three steps in this procedure. First, the lattice is decomposed into isolated blocks such that the Hamiltonian can be written as a sum of the block Hamiltonian and the interblock Hamiltonian. Then, the block Hamiltonian is diagonalized exactly and the low-lying energy eigenstates are kept to build up a projection operator P representing the most important subspace of the original Hilbert space. Finally, the original Hamiltonian is mapped onto the renormalized (or effective) Hamiltonian by utilizing the projection operator ($H_{\text{eff}} = P^{\dagger}HP$) and the relationship between the original and the renormalized coupling constants, i.e., the QRG equations, can be obtained.

By applying the QRG method to 2D systems as proposed in Refs. [37,38], the QPT and the corresponding entanglement on the square lattice can be investigated. Consider a spin-1/2 transverse-field Ising system with Hamiltonian

$$H = -J\sum_{\langle i,j\rangle}\sigma_i^z\sigma_j^z - h\sum_i \sigma_i^x, \tag{1}$$

where $\sigma_i^{\alpha}(\alpha = x, z)$ are the Pauli operators at the site i, and the ferromagnetic exchange coupling $J > 0$ and the transverse field $h \geq 0$ are assumed. The sum $\langle i, j\rangle$ is over all nearest neighbors and periodic boundary conditions are adopted.

According to the scheme developed in Ref. [37], the square lattice is divided into blocks in horizontal and vertical directions, which will be transformed into the effective spins (as shown in Fig. 1). To preserve the symmetry of the lattice, the order of both renormalization directions should be equivalent. The geometric mean of all renormalized coupling strengths, which has been presented in Ref. [38] and yields the renormalized isotropic coupling, will be adopted in this paper. We define the transverse field strength normalized to the exchange interaction as

$$g = h/J. \tag{2}$$

After the RG transformation supplied in Refs. [37,38], the renormalized transverse field strength can thus be obtained of following form:

$$g' = \frac{g^4\left(\left(1+g^2\right)^3\left(4+4g^2+2g^4+g^6\right)\right)^{\frac{1}{4}}}{\left(2+g^2\right)\sqrt{8+8g^2+3g^4+g^6}}. \tag{3}$$