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The sociogeometry of inequality: Part I

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HIGHLIGHTS

- A sociogeometric framework for the measurement of socioeconomic inequality is established.
- The concept of Lorenz sets is introduced and explored.
- A collection of inequality indices based on Lorenz sets is analyzed.

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ABSTRACT

The study of socioeconomic inequality is of prime economic and social importance, and the key quantitative gauges of socioeconomic inequality are Lorenz curves and inequality indices—the most notable of the latter being the popular Gini index. In this series of papers we present a sociogeometric framework to the study of socioeconomic inequality. In this part we shift from the notion of Lorenz curves to the notion of Lorenz sets, define inequality indices in terms of Lorenz sets, and introduce and explore a collection of distance-based and width-based inequality indices stemming from the geometry of Lorenz sets. In particular, three principle diameters of Lorenz sets are established as meaningful quantitative gauges of socioeconomic inequality—thus indeed providing a geometric quantification of socioeconomic inequality.

1. Introduction

The interest of the scientific community in socioeconomic inequality initiated with the pioneering works of Pareto [1], Lorenz [2], Gini [3,4], and Pietra [5]. Nowadays the study of the socioeconomic inequality is a well established pillar of economics, social sciences, and political sciences [6–16]. In the recent years the study of socioeconomic inequality is also drawing considerable interest in econophysics [17–28].

The quantitative evaluation of socioeconomic inequality is usually carried out by measures termed "inequality indices" [6,9,10]. The inequality indices, in turn, stem from quantitative representations of the distributions of wealth in human societies termed "Lorenz curves" [2,8]. The most widely applied inequality index is the well known Gini index [3,4,11].

This is Part I of a pair of papers presenting a novel sociogeometric approach to socioeconomic inequality. In this first part we shift from the notion of Lorenz curves to the new notion of Lorenz sets, and define inequality indices via Lorenz sets. In the second part [29] we further shift from Lorenz curves and Lorenz sets to disparity curves and disparity sets—which set focus on the socioeconomic gap between the rich and the poor. The framework established in this pair of papers yields a collection of new and meaningful inequality indices providing a sociogeometric quantification of socioeconomic inequality.

The pair of sociogeometric papers is written so as to enable an independent reading of each part. The independence comes at the price of some redundancy—which we believe to be a fair price for readability and accessibility. This part is organized

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as follows: we begin with a general presentation of the Lorenz-set sociogeometric framework (Section 2), introduce and analyze distance-based inequality indices (Section 3) and width-based inequality indices (Section 4), order and exemplify the different inequality indices (Section 5), and conclude with a summary of results (Section 6). The quantitative methods applied are detailed in the Methods section (Section 7).

2. Measuring inequality

Considering a given human society, we sample at random a member of this society, and denote by W the personal wealth of this member. We further consider the random variable W to be non-negative valued and to have a positive mean μ, and denote by $f(w)$ $(w \geq 0)$ the random variable's probability density function. In this section we describe various functions corresponding to the random variable W, introduce the notion of the Lorenz set, and describe the notion of inequality indices in terms of the Lorenz set.

A note about notation that will be applied throughout this paper: given a monotone real-valued function $y = \phi(x)$ defined on a real range, we denote by $x = \phi^{-1}(y)$ the corresponding inverse function.

2.1. CDFs

The cumulative distribution functions (CDFs) of the random variable W are given by

$$F(l) = \int_0^l f(w)\, \mathrm{d}w \tag{1}$$

$(l \geq 0)$, and

$$\bar{F}(l) = \int_l^\infty f(w)\, \mathrm{d}w \tag{2}$$

$(l \geq 0)$. Namely, $F(l)$ is the proportion of the society members with personal wealth that is no-larger than the level l, and $\bar{F}(l)$ is the proportion of the society members with personal wealth that is greater than the level l. The CDF $F(l)$ is monotone increasing, the CDF $\bar{F}(l)$ is monotone decreasing, and the two CDFs are coupled by the connection

$$F(l) + \bar{F}(l) = 1 \tag{3}$$

$(l \geq 0)$.

2.2. CMFs

The cumulative mean functions (CMFs) of the random variable W are given by

$$M(l) = \frac{1}{\mu} \int_0^l w f(w)\, \mathrm{d}w \tag{4}$$

$(l \geq 0)$, and

$$\bar{M}(l) = \frac{1}{\mu} \int_l^\infty w f(w)\, \mathrm{d}w \tag{5}$$

$(l \geq 0)$. Namely, $M(l)$ is the proportion of the society's overall wealth held by its members with personal wealth that is no-larger than the level l, and $\bar{M}(l)$ is the proportion of the society's overall wealth held by its members with personal wealth that is greater than the level l. As in the case of the CDFs, the CMF $M(l)$ is monotone increasing, the CMF $\bar{M}(l)$ is monotone decreasing, and the two CMFs are coupled by the connection

$$M(l) + \bar{M}(l) = 1 \tag{6}$$

$(l \geq 0)$.

2.3. Lorenz curves

The Lorenz curve $L(u)$ $(0 \leq u \leq 1)$ of the random variable W has the following meaning: the low (poor) $100u$% of the society members hold $100L(u)$ % of the society's overall wealth. Analogously, the Lorenz curve $\bar{L}(u)$ $(0 \leq u \leq 1)$ of the