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The sociogeometry of inequality: Part II

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HIGHLIGHTS

- A sociogeometric framework for rich–poor disparity is established.
- The concept of disparity sets is introduced and explored.
- A collection of inequality indices based on disparity sets is analyzed.

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ABSTRACT

The study of socioeconomic inequality is of prime economic and social importance, and the key quantitative gauges of socioeconomic inequality are Lorenz curves and inequality indices — the most notable of the latter being the popular Gini index. In this series of papers we present a sociogeometric framework to the study of socioeconomic inequality. In this part we focus on the gap between the rich and the poor, which is quantified by gauges termed disparity curves. We shift from disparity curves to disparity sets, define inequality indices in terms of disparity sets, and introduce and explore a collection of distance-based and width-based inequality indices stemming from the geometry of disparity sets. We conclude with mean-absolute-deviation (MAD) representations of the inequality indices established in this series of papers, and with a comparison of these indices to the popular Gini index.

1. Introduction

This is Part II of a pair of papers studying the sociogeometry of socioeconomic inequality. As stated in the Introduction of the first part: "The interest of the scientific community in socioeconomic inequality initiated with the pioneering works of Pareto [1], Lorenz [2], Gini [3,4], and Pietra [5]. Nowadays the study of the socioeconomic inequality is a well established pillar of economics, social sciences, and political sciences [6–16]. In the recent years the study of socioeconomic inequality is also drawing considerable interest in econophysics [17–28]".

A key tool for quantifying the distributions of wealth in human societies is the method of "Lorenz curves" [2,8]. In turn, this method yields various "inequality indices" — which are quantitative gauges of socioeconomic inequality [6,9,10]. The most widely applied inequality index stemming from the method of Lorenz curves is the well known Gini index [3,4,11]. In Part I [29] we shifted from the Lorenz curves to the novel notion of Lorenz sets, and defined inequality indices via Lorenz sets. This approach gave rise to a collection of new and meaningful inequality indices providing a sociogeometric quantification of socioeconomic inequality.

In the context of socioeconomic inequality the disparity between the rich and the poor is an issue of utmost importance [12–15]. The Lorenz curves quantify the distributions of wealth in human societies from a broad perspective, and do

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not focus specifically on the gap between the rich and the poor. A quantitative focus on the rich–poor gap is facilitated by the method of "disparity curves", which was recently introduced and explored in Ref. [25]. In this part we shift from Lorenz curves to disparity curves, and thereafter from the notion of Lorenz sets to the novel notion of disparity sets. Following the path set in the first part, in this second part we study an analogous collection of inequality indices emanating from the sociogeometric structure of disparity sets. Moreover, in this paper we present mean-absolute-deviation (MAD) representations to all the inequality indices established in both Parts I and II, rank these inequality indices according to their MAD representations, and highlight the marked difference between these inequality indices and the popular Gini index.

This pair of sociogeometric papers is written so as to enable an independent reading of each part. This independence comes at the price of some redundancy — which we believe to be a fair price to pay in order to increase readability and accessibility. In this part we begin with a general presentation of the disparity-set sociogeometric framework (Section 2). Then, in the context of disparity sets, we introduce and analyze distance-based inequality indices (Section 3), and width-based inequality indices (Section 4). Thereafter, the MAD representations of the inequality indices addressed in this pair of papers is presented (Section 5), these inequality indices are discussed and compared to the Gini index (Section 6), and we conclude with a summary of the results established in this pair of papers (Section 7). The quantitative methods applied are detailed in the Methods section (Section 8).

2. Measuring disparity

Consider a given human society whose distribution of wealth is quantified by the pair of Lorenz curves $L(u)$ and $\bar{L}(u)$ $(0 \le u \le 1)$. These Lorenz curves were described in detail in Ref. [29], and are briefly reviewed in Section 2.1. As noted in the introduction, it is often the rich–poor gap – rather than the overall distribution of wealth – that is at the spotlight [12–15]. This rich–poor gap is quantified by the pair of disparity curves, $D(u)$ and $\bar{D}(u)$ $(0 \le u \le 1)$, which we describe in Section 2.2. We then address the ordering of the Lorenz and disparity curves (Section 2.3), shift from these curves to their corresponding Lorenz and disparity sets (Section 2.4), and describe the notion of inequality indices in terms of the Lorenz and disparity sets (Section 2.5). We conclude this section with the illustrative example of the lognormal law (Section 2.6), and with a short description of our agenda regarding the next two sections (Section 2.7).

A note about notation that will be applied throughout this paper: given a monotone real-valued function $y = \phi(x)$ defined on a real range, we denote by $x = \phi^{-1}(y)$ the corresponding inverse function.

2.1. Lorenz curves

The Lorenz curve $L(u)$ $(0 \le u \le 1)$ is defined as follows: the low (poor) $100u\%$ of the society members hold $100L(u)\,\%$ of the society's overall wealth. Analogously, the 'complementary' Lorenz curve $\bar{L}(u)$ $(0 \le u \le 1)$ is defined as follows: the top (rich) $100u\%$ of the society members hold $100\bar{L}(u)\,\%$ of the society's overall wealth. Three key properties of the Lorenz curves are:

- The Lorenz curves increase monotonically from $L(0) = \bar{L}(0) = 0$ to $L(1) = \bar{L}(1) = 1$.
- The Lorenz curve $L(u)$ is convex ($L''(u) \ge 0$), and the Lorenz curve $\bar{L}(u)$ is concave ($\bar{L}''(u) \le 0$).
- The Lorenz curves are coupled by the connection

$$L(u) + \bar{L}(1-u) = 1 \tag{1}$$

$(0 \le u \le 1)$.

The Lorenz curves are a fundamental quantitative tool in economics and the social sciences [8], and are named in honor of the American economist Max Otto Lorenz [2].

2.2. Disparity curves

The disparity curve $D(u)$ $(0 \le u \le 1)$ is defined as follows: the top (rich) $100D(u)\%$ of the society members are in possession of the same aggregate wealth as the low (poor) $100u\%$ of the society members. In terms of the Lorenz curves, the disparity curve $D(u)$ is implicitly given by $L(u) = \bar{L}[D(u)]$. Hence, we obtain that:

$$D(u) = \bar{L}^{-1}[L(u)] \tag{2}$$

$(0 \le u \le 1)$.

Analogously to the disparity curve $D(u)$, the 'complementary' disparity curve $\bar{D}(u)$ $(0 \le u \le 1)$ is defined as follows: the low (poor) $100\bar{D}(u)\%$ of the society members are in possession of the same aggregate wealth as the top (rich) $100u\%$ of the society members. In terms of the Lorenz curves, the disparity curve $\bar{D}(u)$ is implicitly given by $L\left[\bar{D}(u)\right] = \bar{L}(u)$. Hence, we obtain that:

$$\bar{D}(u) = L^{-1}\left[\bar{L}(u)\right] \tag{3}$$

$(0 \le u \le 1)$.