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The evolution of cooperation in spatial prisoner's dilemma games with heterogeneous relationships

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HIGHLIGHTS

- We introduce heterogeneous relationships in evolutionary prisoner's dilemma game models.
- We argue that players tend to preferentially allocate their investments to the intimate friends.
- The presence of diverse relationships guarantees the emergence of cooperation
- Extremely strong investment preference hinders the diffusion of cooperative strategy.

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ABSTRACT

This paper investigates how the introduction of heterogeneous relationships and the subsequent investment preference influences the evolution of cooperation in prisoner's dilemma games. We consider random tie strength values drawn from distributions with different heterogeneity. We show that cooperation is significantly promoted if players preferentially allocate their investments to the friends with strong relationships. The facilitation of cooperation relies mostly on the heterogeneous distribution of relationships and the subsequent investment preference, resulting in strong cooperative allies between good friends and the formation of cooperative clusters around such allies. Moreover, we discover that the investment preference has a sophisticated impact on the evolution of cooperation. Consequently, the level of cooperation will be greatly enhanced by a weak investment preference, but will be significantly depressed if such preference exceeds a critical value. We attribute this finding to the extreme allocation of investments brought by strong preference that hinders the diffusion of cooperative strategy all over the network. Our results suggest that heterogeneous relationships and the subsequent investment preference might have played a crucial role by the evolution of cooperation amongst egoistic individuals.

1. Introduction

The most important issue of all biological systems is cooperation. Cooperative behaviors are required for many levels of organizations ranging from single cells to human groups [1–3]. The formation of human society is based to a large extent on the mechanisms that facilitate cooperative interactions among its members. It is demonstrated that cooperation is promoted in structured populations and on graphs in prisoner's dilemma games (PDG) [4–11]. These efforts recognize the fact

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that player level interactions are not random, but determined by the relationships of the social networks they are embedded in. Since game interactions are implemented through relationships among participants, the nature of these relationships will inevitably pose significant influence on the evolutionary process of PDG. In fact, the relationships in social networks can be depicted by two major properties: structural patterns and tie strengths. The structural patterns of the relationships determine the global picture of a social network, while tie strengths describe the extent of intimacy within each relationship. It is well known that different network structures have different impacts on the evolution of cooperation in PDG, heterogeneous networks are proven to be more facilitative to the emergence of cooperation [4,12–25]. Besides the topology of interaction graph, the strength of links could also be vital. A recent experiment, conducted by Harrison et al., revealed that the behavior of a player in PDG is related to the strength of the relationship between him and his counterpart [26]. Players tend to sacrifice more for the benefits of their intimate friends. Their finding can be regarded as a solid evidence of the existence of heterogeneous influence from tie strength. Therefore, motivated by their observation, we try to reveal how heterogeneous tie strength distributions may influence of the evolution of cooperation in PDG. The conclusion of this study explains the emergence of cooperation from a brand new perspective.

In the classical evolutionary prisoner's dilemma game (PDG) model, each player has two feasible actions: cooperation (C) or defection (D). Both players get R (reward) for mutual cooperation and P (punishment) for mutual defection. A defector exploiting a cooperator gets T (the temptation to defect) and the exploited cooperator gets S (the sucker's payoff). R, P, T, S satisfy following conditions: $T > R > P > S$ and $2R > T + S$. To better illustrate the roles of diverse tie strengths in PDG, in the following we consider an important special case called the "donation game" (DG) [27]. In a DG, each player can cooperate by providing a benefit b to the other player at his or her cost c, with $0 < c < b$. Then, $T = b$, $R = b - c$, $P = 0$, and $S = -c$. The payoff matrix is

	C	D
C	$b - c, b - c$	$-c, b$
D	$b, -c$	$0, 0$

In particular, we wish to extend the scope of stochastic effects on the evolution of cooperation in the spatial prisoner's dilemma game by introducing the heterogeneous distributions of tie strengths in the social network as a predetermined property. This can be realized by introducing heterogeneous tie strength values to each of the relationships that determine the extent of intimacy between each pair of the friends. Importantly, the tie strength values are randomly drawn from different distributions and are determined only once before the game starts. Without loss of generality all tie strength values are between 0 and 1. Presently, we consider tie strength values between player *i* and *j* drawn from $ts_{ij} = \chi^{\lambda}$ with different exponents λ, where χ are uniformly distributed random numbers from the unit interval and λ controls the heterogeneity of the relationships. As an intuitive rule, a cooperator is likely to invest more to their close friends than those with only "nodding relations" and this preference is controlled by a parameter alpha. The cooperator will allocate his investments equally if $\alpha = 0$, indicating that the player has no investment preference. When $\alpha \to +\infty$, the player will give all the investments to the best friend. In the following we investigate how the introduction of heterogeneous relationships, in particular their different distributions and the investment preferences brought by such relationships, affects the evolution of cooperation in spatial grids. We found that the existence of heterogeneous relationships and the subsequent investment preferences is of vital importance to the emergence of cooperation in PDG. We attribute this finding to the formation of strong cooperative allies between good friends, which enable the pairs of cooperative friends to obtain significantly greater payoffs to resist the invasion of defectors. Moreover, we find an interesting phenomenon that the introduction of investment preference has a counterintuitive effect on the evolution of cooperation. The level of cooperation will be greatly enhanced by a small preference α, but will be significantly depressed if α exceeds a critical value. This critical value is positively related to the heterogeneity of the tie strength distribution of the network. However, it is important to note that cooperation will always be preserved if we include this relationship oriented investment preference into the model regardless of the tie strength distribution.

The remainder of this paper is organized as follows: Section 2 describes the spatial donation game model with heterogeneous investment preferences and the properties of tie strength distributions with different exponents λ. Simulation results are discussed in Section 3. In Section 4, we summarize the results and outline some important implications of our findings.

2. The model

We consider a two strategy PDG with players located on a $L \times L$ square lattice with periodic conditions. Each player is allowed to interact with its four neighbors where self-interactions are excluded. In each round, player *x* is allowed to adopt the strategy of a randomly selected friend *y* with a probability $pr_{x \to y}$ proportional to their payoff difference:

Classical donation game model assumes that the ability of a cooperator to invest in a network is proportional to his or her degree [28,29]. A cooperator with *k* friends has a total amount of *kc* investments at the beginning of each round and will allocate the investments equally to all its *k* friends, i.e. each friend receives *c* and produces *b* out of this investment in each round. However, with the introduction of tie strength, we assume that a cooperator will preferentially allocate his investments to his good friends. The cooperator *i* will invest $k_i c \frac{ts_{ij}^{\alpha}}{\sum_m ts_{im}^{\alpha}}$ to his friend *j*. Therefore, the recipient *j* will get $k_i b \frac{ts_{ij}^{\alpha}}{\sum_m ts_{im}^{\alpha}}$ from *i*'s investment. Here, ts_{ij} denotes the strength of the tie between the player *i* and *j*, and *m* runs over all *i*'s friends. α is a tunable parameter controlling the investment preference of the cooperator. If $\alpha = 0$, the model becomes a classical PDG.