Physica A 423 (2015) 61–71

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Fluid flow in a porous tree-shaped network: Optimal design and extension of Hess–Murray's law

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HIGHLIGHTS

- Pressure distribution along a porous network is approached.
- Flow resistance of a porous network is modeled.
- Wall permeability impacts the optimal size ratio of parent and daughter branches.
- Different optimal diameters for lungs' conductive and respiratory zones are explained.
- Hess–Murray's law is justified and extended.

ARTICLE INFO

Article history:
Received 29 October 2014
Received in revised form 15 December 2014
Available online 31 December 2014

Keywords:
Hess–Murray law
Porous-walled tree-shaped networks
Flow resistance
Homothety ratios
Optimal design
Constructal law

ABSTRACT

This paper aims to contribute to the ongoing research on tree-shaped flow structures. Here, we focuses on porous-walled tree-shaped networks, namely the laminar fluid flow. Analytical models are developed for pressure distribution along the porous tree-network and for the hydraulic resistance of the network in terms of geometry of successive vessel segments, number of branches, branching levels and intrinsic permeability of walls. We also rely on constructal design to find important insights regarding the allometric relationships between the sizes of successive vessel segments of tree networks.

Among other results, we show that the flow distribution depends on the aspect ratio of the branching vessels as well as on the wall permeability of vessels. Maximum physical efficiency to connect successive vessel segments is homothetic with a size ratio of $2^{-1/3}$ (Hess–Murray law) only for impermeable tree-networks. Our results indicate that for porous vessels, this homothetic ratio increases with the intrinsic permeability of the vessel wall. This result may help to understand the occurrence of different optimal relationships between the vessel diameters such as in the branching hierarchy of the conductive and respiratory zones of the lungs.

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1. Introduction

Complex flow systems rely on organized networks of directed flows through arrangements of vessels (vasculatures) to guide fluids and solutes from points to finite volumes (and vice-versa). The tree-shaped networks (or tree vascularization) are prevalent in living organisms, geophysics systems and engineering systems [1–7]. In nature, these networks are examples of spontaneous self-organization and self-optimization [1,2,7]. Much effort has been devoted to understand the generation of these configurations both for fundamental reasons and their impact in modern engineering. Therefore, an accurate description of fluid flow behavior in tree-shaped networks is essential to understand the successful design of vascular

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http://dx.doi.org/10.1016/j.physa.2014.12.025
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configuration in plants and animals, as well as the structure and operation optimization of man-made flow trees such as in macro and microfluidic devices [1,8,9].

Distribution systems, such as mammalian circulatory and respiratory trees, develop dichotomous branching trees, where a single vessel (parent, immediately upstream from a vessel bifurcation) divides into two "sister" vessels (immediately downstream after a vessel bifurcation). The respiratory tree, for example, consists of highly branched hollow vessels that become narrower and shorter as they penetrate the deeper part of the lung. Studies performed by Hess [10] and Murray [11] showed that the efficiency of these trees depended on an optimal relationship between the vessel diameters in the branching hierarchy. The reduction of successive vessel segments (from parent to daughter vessels) by a constant factor ($2^{-1/3}$) is usually named as Murray or Hess–Murray law [1,2,5]. This homothety ratio represents the optimum way (i.e., maximum physical efficiency) to connect large and small vessels together to achieve fluid flow with the minimum of power [11] and is of functional significance [2,5]. Bejan et al. [12] rely on the constructal law to establish that also daughter branches are shorter in length compared to the parent vessel (reduction of length by a constant factor $2^{-1/3}$). One observes that lengths and diameters of successive vessel segments change in the same proportion.

Hess–Murray law is supported by a number of empirical studies [13,14]. However, there are some tree networks that seem to show slightly different homothetic ratios. In the respiratory tree, the diameter of acinar airways (respiratory region) seems to fall less steeply than that of conducting airways (first 16 generations of airway) and is thus larger than the prediction from the Hess–Murray law [15,16].

Fluid flow in vessels is described based on the Navier–Stokes equations in their full and simplified forms (see for example, [8,12,17,18]). While air and plasma are Newtonian fluids, blood is associated to non-Newtonian effects. However, in large arteries and veins and in cavities such as the ventricles and atria, non-Newtonian effects cease because of high shear rates and blood behaves essentially as a Newtonian fluid [19,20]. Fluid flow in living organisms is essentially laminar [21] and there are evidences that turbulent flows may even pose health risk [22,23].

Studies of fluid flow in tree-shaped networks are typically performed with tubes having impermeable walls [8,18]. But natural systems do not always fit into this condition [24]. The respiratory tree of mammalian is a good example. From the physiological point of view, this tree can be subdivided into two zones: the conducting zone and the respiratory zone. The trachea and the first 16 generations of airway branches form the conducting zone [25] which distributes air to the deeper parts of the airway tree and no gas exchange occurs through the wall. But, the last seven generations of the respiratory tree comprise the respiratory zone (acinus) [25]. This zone is made up of porous vessels and constitutes the gas exchange unit between blood and air.

This is not a singular example. Although the airway tree is an essential component of breathing process, alone it cannot promote oxygen and carbon dioxide exchange crucial for cells survival [25]. Furthermore, a network of capillaries (vascular tree) surrounds the respiratory zone (acinus) of the tree, and brings blood into close proximity with air within the alveolus. The exchange of oxygen and carbon dioxide is accomplished through porous walls of both alveoli and capillaries. In summary, different overlapped trees (airways, arteries and veins) with porous vessels are closely packed together to ensures a proper exchange of fluids and solutes [26]. In fact, fluid flow along vessels and permeation through the vessels wall often coexist and is observed both in nature and in man-made systems [27–29].

Assuming a uniform permeation through the walls, Berman [30] and Yuan and Finkelstein [31] developed solutions for fluid flow in a rectangular slit and for a cylindrical porous tube, respectively. Pressure and the related leakage may not be uniform along the vessel walls. Therefore, alternative approaches are presented by Refs. [27,32–34]. As far as the author knows, fluid flow in a porous tree-shaped network has not yet been approached.

This study addresses a fundamental issue of distributing a fluid flow in a network of vessels. It investigates the fluid flow in a porous-walled tree-shaped network. Analytical approaches for axial variation in pressure within the vessels and for the fluid flow are presented. The performance of permeable networks designed or not designed according to Hess–Murray law is analyzed.

2. Tree-shaped networks design

Consider a tree-shaped flow network composed by i branches of vessels, from level 0 to level n (Fig. 1). Each vessel, circular in cross section, branches into m daughter branches at the next level. Let the D_i denote the diameter of the vessel at level i and L_i denoting the length of the vessel at level i with $i = 0, 1, 2 \ldots n$. In nature, when vessels bifurcate their size should change by a certain factor, and the ratio of successive diameters and lengths (daughters to parent vessels) can be written as

$$\frac{D_{i+1}}{D_i} = a_D \quad \text{and} \quad \frac{L_{i+1}}{L_i} = a_L \tag{1}$$

where a_D and a_L are scale factors or homothety ratios independent of i [12,18]. Notice that, for laminar flow, according to Refs. [11,12] $a_D = a_L = 2^{-1/3}$ is the optimum way to connect parent vessel branches into daughter vessels. This means that the diameters and lengths are halved after three successive generations.

The relationship between the size of the first vessel (level 0) and the size of the vessel at level i, is given by

$$\frac{D_i}{D_0} = a_D^i \quad \text{and} \quad \frac{L_i}{L_0} = a_L^i. \tag{2}$$

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