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Dynamics of bipartite and tripartite entanglement in a dissipative system of continuous variables

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HIGHLIGHTS

- An analytical criterion was derived for bipartite entanglement in Agarwal bath.
- Entanglement phase diagram constructed showing regions of and without sudden death.
- Using basis transformation, master equations for common bath cases are derived.
- W and GHZ type three modes states are investigated for common and separate baths.
- Common bath mediated inter-mode interactions lead to persistent entanglement.

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ABSTRACT

With the help of the characteristic function and the covariance matrix, we study the effect of a dissipative bath on bipartite entanglement in a system of continuous variables with no direct inter-mode coupling. Under the dissipation induced by an Agarwal bath, the time evolution of entanglement between two oscillators initially in a squeezed state has been examined, and an analytical criterion, as a function of the squeezing parameter s and the bath temperature $\bar{n}$, has been derived for bipartite entanglement at long times, resulting in an entanglement phase diagram comprising of a sudden death region and a no-sudden death region. If $(s, \bar{n})$ lies in the no-sudden death region, there will be sustained bipartite entanglement in the long-time limit. The analysis has been extended to three-mode systems including fully symmetric tripartite states and bisymmetric states, which are invariant with the exchange of two modes (out of the three). If two symmetric modes are propagated in separate baths, the bipartite entanglement between one of the two and the third mode will vanish in a finite time. The disappearance of the entanglement leads to a decrease in cloning fidelities to below the classical value of 0.5. For fully symmetric tripartite states, both the separate- and common-bath cases have been considered. It is found that entanglement vanishes in a finite time in the presence of separate baths, while it persists for a long time in the presence of a common bath.

1. Introduction

In the existing literature [1–7] of quantum information and communication, a tremendous amount of interest has been raised in bipartite and multipartite systems of continuous variables (CV), which are often synonymous to a set of harmonic

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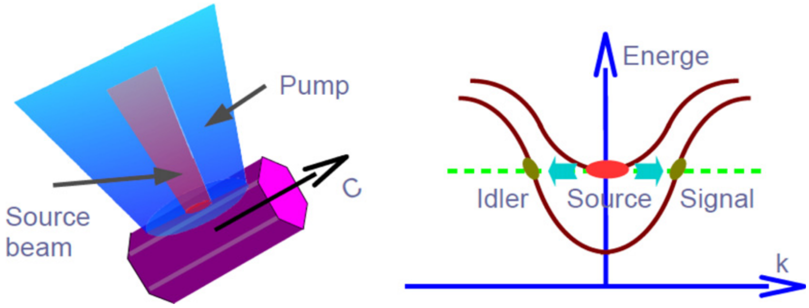

Fig. 1. (Color online) Schematic for the experimental realization on ZnO whispering gallery microcavity (left panel) and the corresponding parametric scattering process for the polariton condensate (right panel).

oscillators. Generating, preserving and quantifying CV entanglement, are some of the issues of fundamental importance, that have been theoretically formulated and experimentally tested [8–11,1,5,12]. For example, a recent effort was focused upon entanglement generation from two remote microscopic systems with no direct interactions, which was claimed to be realized with the assistance of a bosonic bath [12]. On the other hand, technological breakthroughs in quantum optics have renewed interests in the basic subject of entanglement [13–16]. For example, the polariton, an entity mixing light and matter, can be a suitable vehicle for experimental realization of CV entanglement. In particular, Liew and Savona [16] extensively studied the bipartite and multipartite entanglement between polariton modes with the consideration of parametric scattering as an effective interaction among the modes. Thanks to both parametric scattering and repulsive potential induced by the exciton reservoir and the pump field, a strong coupling microcavity in the form of a ZnO whispering gallery structure, was demonstrated to achieve a polariton condensate at room temperature [17–19]. This achievement affords us opportunities to study CV entanglement embedded in a heat bath from a new perspective, which is the main focus of this work.

Generally speaking, a separable density matrix can be represented by a mixture of direct-product states. A bipartite density matrix is separable if it can be written in the form

$$\rho = \sum_i p_i \rho_A^i \otimes \rho_B^i, \tag{1}$$

where $p_i > 0$ and $\sum_i p_i = 1$. An inseparability criterion concerning partial transpose of the density matrix was proposed by Peres [2], and later shown by Horodecki to be a necessary and sufficient condition for inseparability of 2×2 or 2×3 systems [3]. Although the majority of the work in the field is concerned with Hilbert spaces with finite dimensions, interests have also expanded to CV systems. It has been shown by Simon [7] that the Peres–Horodecki criterion is both necessary and sufficient for inseparability in systems of two harmonic oscillators as well. The bipartite entanglement initiated by a common dissipative bath is another interesting topic in this field [20–22,12]. Braustein showed that entanglement can be generated between two qubits in a common bath in thermal equilibrium in the absence of direct inter-qubit coupling, and the entanglement can persist for an arbitrarily long time [4]. Very recently, indirect inter-qubit coupling attributed to a common boson bath has also been discussed in an extension of the spin–boson model [23,24] and in a scenario in which a two-level system is coupled simultaneously to a spin bath and a boson bath [25].

In this work we investigate bipartite and tripartite CV entanglement of oscillator systems coupled to separate as well as common bosonic reservoirs. For an initially squeezed two-mode state, the entanglement is found to persist for a long time in the presence of a common bath, and entanglement in the long time limit depends on the initial squeezing parameter and the bath temperature. For systems of three oscillators, we study the bipartite and tripartite separability of the bisymmetric and fully symmetric states. For the bisymmetric state, the tripartite entanglement is more stable than the bipartite entanglement in the presence of the heat baths. For the fully symmetric case, it is found that the entanglement is sustained if three modes are coupled to a common bath, while it decays to zero if each mode is coupled to an independent bath.

The rest of the paper is organized as follows. In Section 2, we introduce the Brownian motion of two oscillators in a common bath of the Agarwal type with the goal of probing the entanglement evolution of the oscillators. In Section 3, we expand our analysis to tripartite systems and investigate the two-mode and three-mode separability of the bisymmetric and fully symmetric states. Furthermore, the influence of the common bath and independent baths on the dynamical behavior of the entanglement is also considered for the fully symmetric states. Section 4 presents the conclusion of this paper.

2. Two oscillators

In a one-dimensional microcavity with strong coupling, such as the ZnO whispering gallery structure, the polariton condensate undergoes parametric scattering from one transverse electric mode to another [18]. As shown in Fig. 1, two balanced polaritons (signal and idler) could be generated by the parametric scattering, with the pair initially in an entangled state. Due to the strong binding energy of polaritons and the whispering gallery structure, these two polaritons will be trapped