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Simulated annealing algorithm for optimal capital growth

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HIGHLIGHTS

- Extension of the capital growth under multi games case.
- Dynamic optimal capital growth of a portfolio was investigated.
- A general framework that one strives to maximize growth was developed.
- Simulated annealing algorithm was investigated to solve the framework.
- Performance and risk parameter based on real financial data was calculated.

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ABSTRACT

We investigate the problem of dynamic optimal capital growth of a portfolio. A general framework that one strives to maximize the expected logarithm utility of long term growth rate was developed. Exact optimization algorithms run into difficulties in this framework and this motivates the investigation of applying simulated annealing optimized algorithm to optimize the capital growth of a given portfolio. Empirical results with real financial data indicate that the approach is inspiring for capital growth portfolio.

1. Introduction

For an investor, there are two aspects to improve a trading strategy. The first and the most important goal is to achieve a positive expected return. Once this has been achieved, the investor needs to know what percentage of his capital to risk on each trade, this task is often known as asset allocation or position sizing. The principle of asset allocation is maximizing the expected value of the logarithm of wealth after each period, which was originally developed for gambling [1–11]. Breiman's 1961 paper proved that optimal strategy based on log utility will beat any different strategies almost surely in the long run [12].

The gamblers often bet on several games at once, it is interested in blackjack when a player bets on multiple hands or more players share a common bankroll. Simultaneous bets at different tables are independent but at the same table they have a correlation, this should reduce the fraction per hand. Practical applications to long sequences of wagers are especially

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appropriate for the strategy. Hedge fund trading that enters and exits in a few seconds is an application of this. However, when applied to investment with many assets, optimization algorithms such as quadratic programming run into difficulties.

We investigate the problem of dynamic optimal capital growth of a portfolio. A general framework that one strives to maximize the expected log utility of long term growth rate was developed. The original approach ignores many of the constraints faced by real world investors: trading limitations, budget constraint, no short sales, etc. It is very difficult to solve with these constraints. In this study, we investigate applying the simulated annealing algorithm to solve the problem of maximizing long term growth.

Simulated annealing (SA) is a local search technique for combinatorial optimization, such as optimizing functions with multiple variables. Its convergence properties and its ability to escape local optimal have made it a popular technique over the past years [13,14]. SA takes less CPU time than genetic algorithm (GA) when used to solve optimization problems, because it finds the optimal solution using point by point iteration rather than a search over a population of individuals.

This paper is organized as follows. In Section 2, the capital growth strategy was reviewed. A general framework that one strives to maximize the expected log utility of long term growth rate was developed in Section 3. Finally in Section 4, we present the empirical results with real financial data. We show that in consequence, the approach is inspiring for capital growth portfolio.

2. Brief review of the capital growth strategy

We assume that a gambler has found a positive expectation return and he is able to play this game repeatedly, we let the initial bankroll be W_0, and after n iterations the bankroll is W_n. The winning probability is p, the probability of losing is $1-p$. We define the game return as $r_i = (W_i - W_{i-1})/W_{i-1}$, where W_i is the wealth after i turns.

The amount of money he could make depends only on how much he chooses to bet. How much would he bet? Further, suppose the gambler bets fraction f_i of the actual wealth in i turn. After n turns the gambler's wealth equals

$$W_n = W_0 \prod_{i=1}^{n} (1 + f_i r_i). \tag{1}$$

The returns are independent, so the average wealth after n turns can be written as

$$\langle W_n \rangle = W_0 \prod_{i=1}^{n} (1 + E[f_i r_i]). \tag{2}$$

Since the game has a positive expectation, $E[f_i r_i] > 0$ in this situation, in order to maximize $\langle W_n \rangle$, we would maximize $E[f_i r_i]$ at each turn. The optimal strategy is to stake all capital in each trial, however, the probability of ruin is given by $1 - p^n$ and with $p < 1$, $\lim_{n\to\infty}(1 - p^n) = 1$ so ruin is almost sure. Thus maximization of $\langle W_n \rangle$ is not a good criterion for a long run investment.

In 1956, an asymptotically optimal strategy was proposed by Kelly [1], who made use of a quantity G called the exponential rate of growth of the gambler's wealth. Without affecting the results, in our analysis we use natural logarithms

$$G = \lim_{n\to\infty} \frac{1}{n} \ln \frac{W_n}{W_0} \tag{3}$$

as a criterion for investment optimization. Due to the multiplicative character of W_n, G can be rearranged as

$$G = \lim_{n\to\infty} \frac{1}{n} \ln \prod_{i=1}^{n} (1 + f_i r_i) = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \ln(1 + f_i r_i) = \langle \ln(1 + f_i r_i) \rangle. \tag{4}$$

Imagine that a gambler is faced with an infinitely wealthy opponent who will wager bets made on repeated independent trials of a biased coin. For the risk game introduced above is $p \ln(1+f) + (1-p)\ln(1-f)$ which is maximized by the investment fraction

$$f^* = 2p - 1. \tag{5}$$

Moreover, $G(f_c) = 0$ so we get unique number $f_c > 0$, where $0 < f^* < f_c < 1$. The gambler's wealth will exceed the initial value when f is chosen in the internal $(0, f_c)$. But, if $f > f_c$, the ruin is almost sure. In order to maximize wealth, we should maximize $E[\ln(1 + f_i r_i)]$ by choosing the optimal fraction f^* at each trial although the probabilities change from one trial to the next.

The gambler introduced here follows a different criterion from the classical gambler. Because of the logarithm which is additive in repeated bets and to which the law of large numbers applies. At every bet he maximizes the expected value of the logarithm of his capital. The criterion asymptotically maximizes the expected growth rate of wealth, which is often called the capital growth strategy.

A criticism applied to the strategy is that capital is not infinitely divisible but multiples of a minimum unit. If the minimum bet allowed is small relative to the gambler's initial capital, the probability of ruin in the standard sense is negligible. In the security markets, the minimum unit can be as small as desired.