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# Effect of optimal estimation of flux difference information on the lattice traffic flow model

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## HIGHLIGHTS

- A new lattice model considering the optimal estimation flux difference information has been developed.
- The linear stable condition of the new model is derived.
- The mKdV equation is obtained and solved to describe the traffic jamming transitions.
- The results show that the new consideration can improve the stability of traffic flow.

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ABSTRACT

In this paper, a new lattice model is proposed by considering the optimal estimation of flux difference information. The effect of this new consideration upon the stability of traffic flow is examined through linear stability analysis. Furthermore, a modified Korteweg–de Vries (mKdV) equation near the critical point is constructed and solved by means of nonlinear analysis method, and thus the propagation behavior of traffic jam can be described by the kink–antikink soliton solution of the mKdV equation. Numerical simulation is carried out under periodical condition with results in good agreement with theoretical analysis, therefore, it is verified that the new consideration can enhance the stability of traffic systems and suppress the emergence of traffic jams effectively.



## 1. Introduction

Traffic jam has become a serious problem in modern societies and various important studies have been conducted to investigate the properties of traffic jam. Traffic model [1–5], as an effective tool for revealing the complex mechanisms of traffic congestion, has received increasing attention from researchers. In 1998, Nagatani [6] firstly proposed a lattice model to describe the collective dynamical evolution of traffic congestions on freeway from macroscopic aspect. Using this model, he derived the modified Korteweg–de Vries (mKdV) equation to describe the transition process of traffic jams near the critical point. Thereafter, to describe the nature of traffic jams more realistically, a number of extended lattice models [7–21] were successively constructed on single lane by incorporating different factors such as backward effect [8], driver's anticipation effect [9], lateral effect of lane width [10], driver's delay response [12], and so on. Beyond the lattice model for single lane, two-lane lattice model with lane changing behaviors also has attracted the interests of physicists and engineers. In two-lane framework, Nagatani [22] first presented a two-lane lattice model for traffic flow. Starting from this original two-lane

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lattice model, some modified two-lane lattice models [9,23–28] were proposed by incorporating different factors. It is easy to verify that these lattice models mentioned above can describe the macro trends of the dynamic traffic and suppress the jams of the traffic system to some degree.

Due to the demand of public, it becomes urgent in modern society to maximize the throughput of cars on freeways and suppress traffic jams. Therefore, we concentrate on the enhancement of traffic stability to prevent the traffic congestion with as much traffic information as possible. In particular, the estimation information about difference between expected optimal flux and current actual flux (called as optimal estimation of flux difference information, OEFDI) may have important influence on traffic flow. In fact, in real traffic, a driver often estimates the difference between expected optimal fluxes and observed actual flux, then adjusts his/her velocity to achieve the optimal state as soon as possible according to the values of OEFDI. Hence, it is closer to reality to incorporate this new consideration into lattice traffic flow models. However, to our knowledge, the effect of OEFDI has not been explored in the lattice models up to now. Thus, we do not know whether or not this new consideration can stabilize the traffic flow and smooth the fluctuation of traffic jams. This motivates us to develop a new lattice model by incorporating the effect of OEFDI.

In view of the reasons mentioned above, a new lattice model is proposed in this paper to incorporate OEFDI factors into lattice models on a single lane highway, and then linear stability analysis is conducted to investigate the effect of this new consideration on the stability of traffic flow. Moreover, through nonlinear analysis, the evolution of traffic jam near the critical point is also explored and the corresponding mKdV equation is obtained to describe the jamming transition. Finally, numerical simulation is carried out to demonstrate the validity of our analysis.

## 2. Models

In 1998, Nagatani [6] firstly proposed a lattice model to describe the collective properties of traffic flow on a freeway. The dynamical equation is described as follows:

$$\partial_t \rho_j + \rho_0(\rho_j v_j - \rho_{j-1} v_{j-1}) = 0 \tag{1}$$

$$\rho_j(t+\tau)v_j(t+\tau) = a\rho_0 V(\rho_{j+1}) \tag{2}$$

where $\rho_0$ denotes the average density; $\rho_j(t)$ and $v_j(t)$ denote the local density and local velocity on site $j$ at time $t$ respectively; $\tau = 1/a$ denotes the delay time, in which $a$ is the sensitivity of a driver; $V(\rho_{j+1})$ is the optimal velocity function [22] that is assumed to be:

$$V(\rho) = \tanh\left(\frac{2}{\rho_0} - \frac{\rho}{\rho_0^2} - \frac{1}{\rho_c}\right) + \tanh\left(\frac{1}{\rho_c}\right) \tag{3}$$

where $\rho_c$ is the critical density. Function $V(\rho)$ has a turning point (inflection point) at $\rho = \rho_c$, when $\rho_0 = \rho_c$.

Based on the lattice model, it is convenient to analyze the evolution of jamming transition in traffic flow and derive the corresponding equation of density wave. Thereafter, in light of the previous work, many improved lattice models have been proposed considering other factors as mentioned in Section 1. However, in these existing lattice models, the effects of difference between expected optimal flux and current actual flux have not been involved yet. In fact, drivers often estimate the information of difference between the observed actual flux and the expected optimal flux (optimal estimation of flux difference information, OEFDI), then adjust the speed of their cars to the optimal state accordingly. Considering this, we develop a new lattice model by introducing OEFDI effect as follows,

$$\rho_j(t+\tau)v_j(t+\tau) = \rho_0 V(\rho_{j+1}(t)) + k\Delta Q_j \tag{4}$$

where the term $\Delta Q_j = \rho_0 V(\rho_0) - \rho_j(t)v_j(t)$ is the optimal estimation of flux difference information (OEFDI) between the observed actual flux and the expected optimal flux on site $j$ at time $t$, $k$ denotes the response sensitivity to the stimulus value of OEFDI. The idea of the new model is that the flux $\rho_j(t+\tau)v_j(t+\tau)$ on site $j$ at time $t+\tau$ is determined by the optimal flow $\rho_0 V(\rho_{j+1}(t))$ and the OEFDI information $\Delta Q_j = \rho_0 V(\rho_0) - \rho_j(t)v_j(t)$ on site $j$ at time $t$. When $k = 0$, our model reduces into Nagatani's lattice model [6].

By eliminating the speed $v$ in Eqs. (1) and (4), the density equations for the new model are obtained as follows:

$$\rho_j(t+2\tau) - \rho_j(t+\tau) + \tau\rho_0^2[V(\rho_{j+1}(t)) - V(\rho_j(t))] + k\{\rho_j(t+\tau) - \rho_j(t)\} = 0. \tag{5}$$

## 3. Linear stability analysis

For all types of lattice hydrodynamic models, the performance of traffic system stability, undoubtedly, is an important issue. The steady state of traffic system is defined as the uniform traffic flow with constant density $\rho_0$ and optimal velocity $V(\rho_0)$ in lattice model, hence the solution of the homogeneous traffic flow is given as:

$$\rho_j(t) = \rho_0, \qquad v_j(t) = V(\rho_0). \tag{6}$$