Physica A 463 (2016) 63–76

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

On the source of stochastic volatility: Evidence from CAC40 index options during the subprime crisis

Skander Slim

LaREMFiQ-IHEC, University of Sousse, B.P. 40 Route de la ceinture, Sahloul III, Sousse 4054, Tunisia

HIGHLIGHTS

- Time-changed Lévy processes are used to model the underlying index price.
- The stock index return is driven by three distinct sources of stochastic volatility.
- All three sources of stochastic volatility are needed for index option pricing.
- A negative risk premium is attributed to both diffusion and downside jump volatility.
- A positive risk premium generated by upside jump volatility is uncovered.

ARTICLE INFO

Article history:
Received 4 October 2015
Received in revised form 5 January 2016
Available online 14 July 2016

Keywords:
Equity index options
Stochastic volatility
Time-changed Lévy process
Volatility risk premium
Square-root unscented Kalman filter

ABSTRACT

This paper investigates the performance of time-changed Lévy processes with distinct sources of return volatility variation for modeling cross-sectional option prices on the CAC40 index during the subprime crisis. Specifically, we propose a multi-factor stochastic volatility model: one factor captures the diffusion component dynamics and two factors capture positive and negative jump variations. In-sample and out-of-sample tests show that our full-fledged model significantly outperforms nested lower-dimensional specifications. We find that all three sources of return volatility variation, with different persistence, are needed to properly account for market pricing dynamics across moneyness, maturity and volatility level. Besides, the model estimation reveals negative risk premium for both diffusive volatility and downward jump intensity whereas a positive risk premium is found to be attributed to upward jump intensity.

1. Introduction

The key role of jumps and stochastic volatility in modeling stock returns and cross-sectional option prices is widely documented in the literature. Stochastic volatility models are specifically designed to capture silent features of volatility such as randomness and persistence. Among one-factor stochastic volatility models, the most popular and easy to implement is the Heston [1] model, which allows return volatility itself to follow a separate diffusion process. However, empirical evidence points strongly to the inadequacy of one-factor volatility models to fully capture the volatility dynamics and the volatility smile (see for instance, Refs. [2–4]). Extensions may be achieved by adding a second source of uncertainty leading to multi-factor stochastic volatility models, even though they often come at the cost of losing some, if not all, analytical tractability. In these models, the mean reversion of volatility is captured using two stochastic processes one with a slow mean reversion and the other with a fast mean reversion property which allow for more flexibility in modeling the volatility term structure. Examples for pure diffusion multi-factor models include Christoffersen et al. [5], Fouque et al. [6], Fouque

E-mail address: slim_skander@yahoo.fr.

http://dx.doi.org/10.1016/j.physa.2016.06.136
0378-4371/

and Han [7]. Models with both stochastic volatility and stochastic jump intensity are proposed by Bakshi et al. [8], Bates [9], Christoffersen et al. [10], Duffie et al. [11] and Eraker [12], among others.

The theoretical framework proposed by Carr and Wu [13] has established Lévy processes as an attractive alternative to the affine jump-diffusion class of Duffie et al. [11] for modeling asset price dynamics. Hence, one tractable way of generating stochastic volatility is to apply separate time changes to multiple Lévy components as building blocks representing different sources of return variation. Huang and Wu [14] analyze the pricing performance of a variety of time-changed Lévy processes and find that market volatility has two components, one arises from diffusion shocks and the other from the arrival rate of jumps. Carr and Wu [15] use two-factor models for foreign exchange rates options. Their specifications generate stochastic skew by applying separate stochastic time-change to the right skewed and left skewed Lévy components. Recent empirical evidences in support of independently specified intensity processes are presented in Refs. [16,10,17].

Throughout this study, we assume that the underlying price dynamics follows a diffusion-extended CGMY process [18] which is rich enough to encompass different jump structures. We specify a three-factor state dynamics for return volatility by applying stochastic time changes to three Lévy components. One activity rate controls the intensity of diffusion and hence the normal innovation component and the other two activity rates control the intensity of positive and negative jump components separately. The variation of the three activity rates over time generates variation in the relative proportion of the three return components. This specification is related to time-changed models estimated by Carr and Wu [15] as it accommodates stochastic skew in the index return dynamics, but it differs in the sense that the diffusion component runs on its own random time clock. Thereby, our multi-factor model allows for potentially different time variation in the arrival rates of small and large events. Furthermore, the separate treatment of negative and positive jump intensities embeds stochastic skew in index return dynamics as return volatility responds differently to the arrival rates of signed jumps [19]. For instance, during a time of bear market conditions, with high negative jump arrival rate, the business time flows faster and downward jumps occur at an increased rate. That is, one expects to observe a higher contribution to the aggregate market volatility from negative jumps than that from positive jumps. Conversely, during calm periods, return volatility is expected to be mainly driven by the stochastic diffusion component. Hence, our specification of separate intensity processes allows for distinct sources of uncertainty that would be priced differently by the market [20].

Based on the most general three-factor model, labeled SVDJJ, we consider three nested models. The most restrictive model (SVD) has a stochastic diffusion component with constant jump intensity. The SVJJ model makes jump arrival rates time-varying by specifying separate intensity processes for the upward and downward jumps while the instantaneous variance of the diffusion component is constant. In the SVDJ model, stochastic volatility is driven by independent stochastic time changes applied to both the diffusion component and jumps. Under this specification, the arrival rate of upward and downward jumps is driven by the same intensity process. We investigate to what extent multi-factor models can improve cross-sectional fit of option prices on their nested lower-dimensional counterparts. In particular, we address the following questions. Which Lévy component contributes significantly to stochastic volatility and how its contribution varies over time? How does the market prices volatility risk?

We use daily option prices on the CAC40 index from December 3, 2007 to April 30, 2009. We cast the models into a state-space representation and estimate the model parameters using the maximum likelihood method. An efficient implementation of the unscented Kalman filter (i.e., the square-root unscented Kalman filter) allows the extraction of latent volatility factors and yields dynamically consistent models.[1]

Our empirical results emphasize the importance of both stochastic diffusive volatility and time-varying jump intensities in pricing CAC40 index options. In-sample performance test reveals strong support for the SVDJJ and SVDJ models whereas the SVJJ model generates the worst performance. This finding suggests that the performance ranking corresponds to an increase in the number of volatility factors only when diffusive stochastic volatility is incorporated. Our richly parameterized SVDJJ model significantly improves on the two-factor SVDJ model, in-sample as well as out-of-sample, thereby indicating the benefit of disentangling volatility variation generated by negative jumps from that generated by positive jumps. We find that most of return volatility variation is explained by the arrival rate of negative jumps during the full unfolding of the subprime crisis, while diffusive stochastic volatility has its largest impact in relatively calmer periods.

The estimation results also show that return volatility generated from diffusion and jump components show different risk-neutral dynamics. The diffusion-induced as well as negative jump-induced volatilities exhibit higher persistence while the positive jump-induced volatility shows much faster mean reversion rate. As a result, the behavior of short-term options is influenced more by the randomness in the arrival rate of positive jumps, but the behavior of long-term options is mostly influenced by randomness from both the diffusive movements and the arrival rate of negative jumps.

Our modeling approach enables us to shed light on the nature of the volatility risk premium embedded in the cross section of options prices. The difference in persistence between risk-neutral and times-series dynamics of the extracted volatility factors captures the market price of volatility risk. Within our three-factor SVDJJ model, the volatility risk premium is decomposed into a premium for volatility risk associated with normal shocks, a premium associated with positive jump shocks and a premium related to negative jump shocks. We find that whereas a negative risk premium is attributed to diffusive volatility variation, the market prices differently volatility risk associated with the two jump components. A

[1] In a dynamically consistent model, the parameters that are allowed to vary over time are converted into state variables, and their dynamics is priced by the market.