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An experiment on Lowest Unique Integer Games[☆]

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HIGHLIGHTS

- We provide first experimental results on three and four player lowest unique integer games.
- We find subjects self-organize themselves into different behavioral classes.
- More than 1/3 of our subjects behaved according to the symmetric mixed strategy Nash equilibrium.
- Those subjects who kept choosing the same number won more frequently than other who did.

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ABSTRACT

We experimentally study Lowest Unique Integer Games (LUIGs) to determine if and how subjects self-organize into different behavioral classes. In a LUIG, N ($\geq$3) players submit a positive integer up to M and the player choosing the smallest number not chosen by anyone else wins. LUIGs are simplified versions of real systems such as Lowest/Highest Unique Bid Auctions that have been attracting attention from scholars, yet experimental studies are scarce. Furthermore, LUIGs offer insights into choice patterns that can shed light on the alleviation of congestion problems. Here, we consider four LUIGs with $N = \{3, 4\}$ and $M = \{3, 4\}$. We find that (a) choices made by more than 1/3 of subjects were not significantly different from what a symmetric mixed-strategy Nash equilibrium (MSE) predicts; however, (b) subjects who behaved significantly differently from what the MSE predicts won the game more frequently. What distinguishes subjects was their tendencies to change their choices following losses.

1. Introduction

We ran laboratory experiments to study the manner in which subjects play Lowest Unique Integer Games (LUIGs) and if and how they self-select themselves into different behavioral groups.

In a LUIG, N ($\geq$3) players simultaneously submit a positive integer up to M. The player choosing the smallest number that is not chosen by anyone else is the winner. In cases where no player chooses a unique number, there is no winner. For

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instance, suppose there is a LUIG with $N = 3$ and $M = 3$. There are three players, A, B, and C, who each submit an integer between 1 and 3. If the integers chosen by A, B, and C are 1, 2, and 3, respectively, then A wins the game. If the integers chosen by A, B and C are 1, 1, and 2, respectively, then C is the winner. And, as noted, if all of them choose the same integer, there is no winner.

LUIGs are highly simplified versions of real systems such as the Swedish lottery game Limbo where the player who chooses the lowest unique number wins a cash prize [1] and Lowest/Highest Unique Bid Auctions (LUBA/HUBA) like the Auction Air or Juubeo websites where players bid, by paying a fixed participation fee, on prizes such as electronic devices or jewelry and either the lowest or highest unique bid wins. These systems differ from LUIGs in that the exact number of players (or participants) is not known when participants are deciding which integer to choose or how much to bid. In addition, unlike with LUIGs or the Swedish lottery, in LUBA/HUBA scenarios, a winner has to pay the amount s/he bids in exchange for the item being auctioned.[1]

These types of real systems have been attracting much attention recently from scholars of various disciplines [2–13]. While the studies mentioned investigate these related systems theoretically and empirically, experimental studies on LUIGs and related systems are still scarce. Östling et al. [1], Otsubo et al. [9], and Chmura and Güth [14] are three experimental studies that are closely related to ours. Östling et al. [1] experimentally study a version of the Swedish lottery in which subjects were not informed of the exact number of players in the game. Otsubo et al. [9] experimentally study a version of LUBA/HUBA in which subjects were informed of the number of players, N, and the strategy space, $\{1, 2, .., M\}$, for $(N, M) \in \{(5, 4), (5, 25), (10, 25)\}$. But unlike our experiment in which the winner receives a fixed payoff regardless of the number chosen, the winner's payoff in Ref. [9] depends on the number he or she has chosen. Chmura and Güth [14] use experiments to study a three-player minority game. In a minority game, three or more odd-numbered players choose one of two options. Thus, a three-player minority game is structurally equivalent to a LUIG with $N = 3$ and $M = 2$. The minority game originates from the "El Farol Bar" problem [15] which is based on people independently deciding whether to go to a popular bar or not; if too many people independently choose to go to the bar, it will be too crowded and those who stayed at home are better off than those at the bar. The opposite is true when the bar is not too crowded. This problem has been extensively studied both by economists and physicists. See, among others, Challet and Zhang [16,17] for theoretical analyses, and Bottazzi and Devetag [18,19]; Chmura and Pitz [20]; Devetag et al. [21]; Linde et al. [22]; Płatkowski and Ramsza [23] for experimental studies with larger numbers of players. The experimental method of Chmura and Güth [14] is, however, different from ours in that they directly elicit a mixed strategy (a probability distribution over two choices) from each subject. In our experiment, each subject submits a pure strategy (i.e., an integer).

Furthermore, LUIGs are fertile ground for experimental study because unlike minority games where multiple players can win the game, LUIGs consider a situation with an extreme congestion effect because at most one player can win. Games with a congestion effect have applications for problems such as the choice of traffic routes [24] that are known to generate an interesting phenomenon called the Braess paradox, a situation exemplified by improvements in a road network actually worsening traffic problems [25,26]. In situations with congestion effects, an interesting question is how players learn to either consciously or unconsciously coordinate their behavior so that inefficiencies due to congestion are reduced. Hanaki et al. [27] theoretically examine this question in what they call "a parking problem". In their "parking problem", available options are clearly ordered in terms of their desirability which is the same for all players. Since each option can be taken by at most one player, when more than one player chooses the same option, only one of them obtains it. By applying a reinforcement learning model, Hanaki et al. [27] show that ex ante homogeneous players separate themselves into "lucky" and "unlucky" ones. They show that some "lucky" players learn to choose "better options" all the time because their early successes in doing so reinforced these options as the right choice, but other "unlucky" players learn to choose "worse options" all the time because in the early stages of their learning processes they failed to obtain a good outcome when they had actually chosen the "better option". However, it is precisely because of the existence of the latter players who have settled into choosing "worse options" that the former group continues to have a positive experience from doing what they do, namely, choosing "better options".

In this paper, we aim to study such self-organization among players experimentally. In particular, we wish to gain insight into whether and how subjects separate themselves into different behavioral classes in situations with an extreme-congestion effect such as LUIGs. Recall that in LUIGs, the winning payoff is the same regardless of the number the winner has chosen. We expect this feature, which is absent in the LUBA or HUBA scenarios mentioned above, will facilitate the self-organization of subjects that we are interested in. In addition, by eliminating the round-by-round fluctuations in the number of players, we can focus on subjects' behavior in a stable (at least in terms of number of players) environment, which should again favor self-organization. Thus, we believe LUIGs offer a good starting point for studying self-organization among players.

We found that some subjects kept choosing the same number, while others switched their choices often. We also found that the first type of subject won the game more frequently than the latter type of subject. However, it should be noted that a strategy of sticking to the same number pays off in a LUIG only when other players do not employ the same strategy. Thus, it is the existence of the latter type of subjects that benefits the strategy employed by the former types. We also found that

[1] There are other differences depending on the auction site. For example, in a HUBA on the Juubeo website, participants can place multiple bids by paying a small fee for each additional bid and they are immediately informed whether their current bid is the current winning bid.