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An evolutionary strategy based on partial imitation for solving optimization problems

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HIGHLIGHTS

- We introduce a novel method for combinatorial optimization problems.
- Social imitative mechanisms may allow to find the solution of the TSP.
- Few agents, that partially imitate each other, are able to converge towards the optimal solution of combinatorial optimization problems.

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ABSTRACT

In this work we introduce an evolutionary strategy to solve combinatorial optimization tasks, i.e. problems characterized by a discrete search space. In particular, we focus on the Traveling Salesman Problem (TSP), i.e. a famous problem whose search space grows exponentially, increasing the number of cities, up to becoming NP-hard. The solutions of the TSP can be codified by arrays of cities, and can be evaluated by fitness, computed according to a cost function (e.g. the length of a path). Our method is based on the evolution of an agent population by means of an imitative mechanism, we define 'partial imitation'. In particular, agents receive a random solution and then, interacting among themselves, may imitate the solutions of agents with a higher fitness. Since the imitation mechanism is only partial, agents copy only one entry (randomly chosen) of another array (i.e. solution). In doing so, the population converges towards a shared solution, behaving like a spin system undergoing a cooling process, i.e. driven towards an ordered phase. We highlight that the adopted 'partial imitation' mechanism allows the population to generate solutions over time, before reaching the final equilibrium. Results of numerical simulations show that our method is able to find, in a finite time, both optimal and suboptimal solutions, depending on the size of the considered search space.

1. Introduction

In the last decades, several evolutionary algorithms [1–4] have been proposed to solve a wide range of optimization problems [5–7]. Evolutionary strategies are generally based on a common scheme: a set of solutions is randomly generated then, according to specific rules, it evolves evaluating the quality of solutions by a cost/gain function. Optimization problems may have a continuous or a discrete spectrum of solutions, defined search space. Here, we focus on discrete cases, usually referred to as combinatorial optimization problems [8]. The latter may have a huge amount of feasible solutions, identified as the minima of the cost function (or as the maxima of a gain function) of the considered problem. The size of the search space is often too large to adopt an algorithm that explores all solutions to identify the optimal one. One of the aims of quantum

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computing [9] is to implement algorithms that are able to span, in a finite time, huge search spaces not accessible by classical algorithms. However, in the meanwhile, strategies based on the classical nature of computing play a prominent role. In this scenario, evolutionary methods such as genetic algorithms [1], and likewise other nature inspired strategies [4,10], allow to define heuristics that compute, in short time, a good suboptimal solution. As mentioned, in mathematical terms, an optimal solution corresponds to the global minimum/maximum of a cost/gain function. Therefore, the search space of these problems can be represented by an n-dimensional cost/gain function, having a continuous or a discrete domain. In this work, we focus on the Traveling Salesman Problem (TSP hereinafter) [11], a well-known problem with a discrete search space that, under opportune conditions, become *NP*-hard. Briefly, the TSP is based on a traveler who wants to visit a list of cities reducing the relative cost as far as possible (e.g. the time or other resources). Hence, optimizing algorithms (e.g. Refs. [11,12]) attempt to compute the best path among the listed cities. Here, the best path is the one that optimizes a function (i.e. minimizing a cost or maximizing a gain). Increasing the number of cities, the number of feasible solutions diverges to very high values. Thus in extreme conditions even to find a good local minimum (i.e. a sub-optimum) becomes a challenging task. In particular, if every city can be visited only once, and if all cities are directly connected, the amount of feasible solutions corresponds to the factorial of the number of cities. For instance, with only 10 cities there are 10! possible paths, i.e. more than $3.6 \cdot 10^6$. The solutions of the TSP can be codified by array structures containing an ordered list of cities, so that each entry of the array corresponds to a city. We aim to face the challenge of finding the optimal (or a good suboptimal) solution of the TSP by means of an agent population, whose evolution is based on an imitative mechanism described below. Imitation processes [13] have been thoroughly investigated in several scientific fields, from social psychology [14] to sociophysics [15,16] and quantitative sociology [17]. For example, a wide variety of Ising-like models [18–20], such as the voter model [21], allow to represent imitative mechanisms in the context of opinion dynamics [16,22,23], evolutionary games [24–27], and many other domains. In the proposed model, agents of a population are provided with randomly generated solutions of a TSP. As stated above, the fitness of each solution is computed by a cost function. Then, through interacting, agents can imitate better solutions (according to fitness). Now, note importantly that the implemented imitative mechanism is partial, hence defined 'partial imitation', since it entails that each agent copies only one entry of a solution better than its own. Thus, since a solution is codified by an array, 'partial imitation' involves copying the value contained in just one entry. In doing so, as we explain later, the agent population can generate solutions during its evolution and, finally, converges towards a common one (i.e. shared by all agents).

A statistical physics overview

At this point, note that our population behaves like a spin system undergoing a cooling process. This implies that, at equilibrium, an ordered phase will be reached, with all spins aligned in the same direction. In particular, spin systems show order–disorder phase transitions [19] driven by temperature: at high temperatures they form a disordered paramagnetic phase, while at low temperatures (i.e. lower than a critical one, also defined as 'Curie temperature') an ordered ferromagnetic phase emerges. Order–disorder phase transitions are well studied in Ising-like models, for instance to analyze the evolution of two-opinion systems [28] in the presence of external influences (e.g. media), particular behaviors (e.g. conformity or stubbornness), or other attributes (e.g. gain) provided to agents. As a result, simple two state models ($\sigma \pm 1$), can be studied analytically using the Curie–Weiss model formalism [19,29], so that the phenomenon of order–disorder transitions is well described. According to thermodynamics [19], the equilibrium of a system corresponds to the minimum of its free energy F. In the Curie–Weiss model we have a paramagnetic phase characterized by one minimum of F, where both states (i.e. $\sigma = \pm 1$) coexist, and a ferromagnetic phase with two possible minima of F, corresponding to $\sigma = +1$ and to $\sigma = -1$. Therefore, in the ferromagnetic phase all spins are aligned in one direction. Here, the system magnetization M [18] is a useful order parameter that allows to directly evaluate the nature of an equilibrium. It is defined as

$$M = \frac{1}{n} \sum_i s_i \tag{1}$$

with n number of spins. A more complex scenario arises when studying the dynamics of spin glasses [30] (e.g. neural networks [31]). In particular, spin glasses are systems characterized by a large number of free energy minima at low temperatures. Because of the topology of spin interactions several configurations, usually referred to as patterns [32], can be reached at equilibrium. In these systems, the concept of order–disorder phase transition is a little more complex. In particular, an ordered state does not correspond to the simple series of aligned spins, but to a particular pattern ϵ, e.g. $\epsilon = [+1, +1, -1, +1, -1]$. As discussed before, system magnetization makes it possible to detect the nature of a system state [33]. Thus values of M close to zero indicate that the system is in a disordered (paramagnetic) phase, while values of $M = +1$ and $M = -1$ mean the system is in an ordered (ferromagnetic) phase. Now, by using the so-called Mattis gauge, we can define a magnetization m that evaluates whether a system is ordered according to a particular pattern. The Mattis magnetization reads

$$m = \frac{1}{n} \sum_i \epsilon_i s_i \tag{2}$$

with ϵ_i value in the ith position of the pattern, s_i value of the spin in the same position of a signal S of length n. As we can observe, when spins are perfectly aligned with a pattern ϵ, the Mattis magnetization is 1. Note that our population cannot