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Squared sine logistic map

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HIGHLIGHTS

- We propose a perturbated logistic map.
- We observe different routes to chaos.
- We observe an unlimited sequence of center-saddle bifurcations.
- The Lyapunov diagrams present qualitative differences for the routes to chaos.
- The bifurcations present robust scaling features.

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ABSTRACT

A periodic time perturbation is introduced in the logistic map as an attempt to investigate new scenarios of bifurcations and new mechanisms toward the chaos. With a squared sine perturbation we observe that a point attractor reaches the chaotic attractor without following a cascade of bifurcations. One fixed point of the system presents a new scenario of bifurcations through an infinite sequence of alternating changes of stability. At the bifurcations, the perturbation does not modify the scaling features observed in the convergence toward the stationary state.

1. Introduction

The logistic map is possibly the most studied one-dimensional map since it is very simple and produces fundamental results on non-linear dynamics [1]. The map allows one to understand the concepts of bifurcations [2], crisis [3,4], route to chaos [5], among other concepts [6–10]. The map is essentially a quadratic function of the type, $f(x_n) = x_{n+1} = rx_n(1 - x_n)$ where r is a control parameter and the map iterations are mapped on the limited range $x \in [0, 1]$ while $r \leq 4$. From this value a boundary crisis occurs [3,4], the chaotic attractive set is destroyed and the iterations escape from this range and go to infinity. There are two fixed points, x^*, for this map, one at $x_1^* = 0$ and another r-dependent given by $x_2^* = (1 - 1/r)$. A transcritical bifurcation is observed at $r = 1$, where the fixed points x_1^* and x_2^* exchange their stability. For $r > 1$, the fixed point x_1^* becomes unstable. For $r = 3$ a first period doubling bifurcation happens involving x_2^*, which defines a route of flip bifurcations, known as Feigenbaum scenario [8,9], toward a chaotic dynamics. For $r = 4$, the fixed point x_1^* touches the chaotic attractor at $x = 0$, producing a boundary crisis, hence destroying the attractor. After the destruction, the chaotic attractor is replaced by a chaotic transient which is characterized by a power law [11]. In order to search for new bifurcations or new mechanisms to chaos, we perturb the logistic map with a squared-sine perturbation. In Section 2 we present our model and the numerical results, including plots of the bifurcation diagram, the associated Lyapunov exponents

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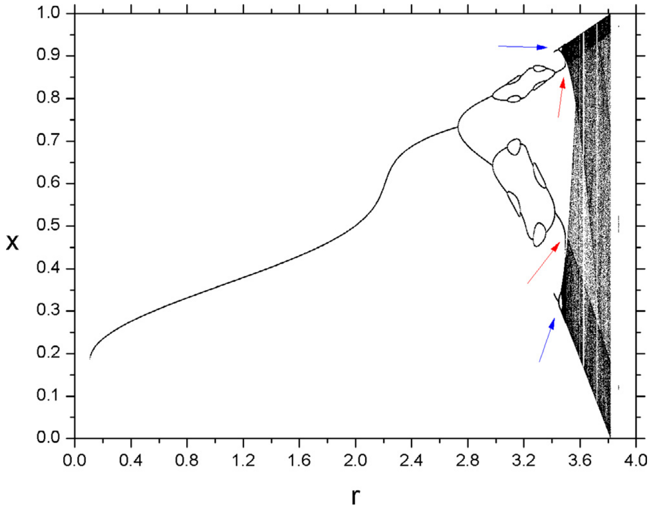

Fig. 1. Bifurcations diagram for the squared sine logistic map. The blue arrows indicate the period doubling cascade toward the chaos, while the red arrows indicate another route to chaos.

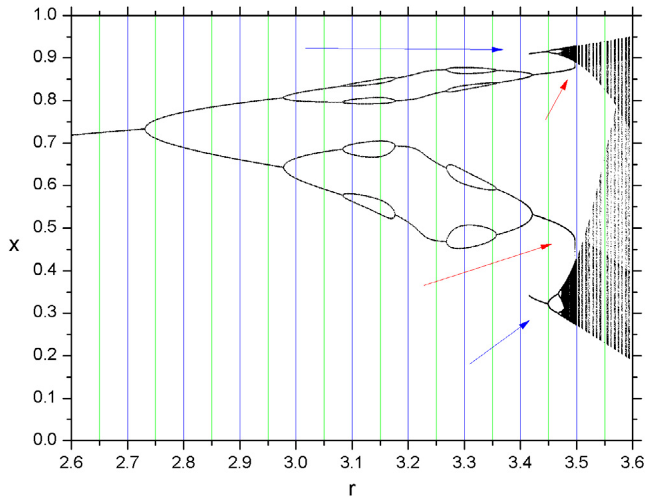

Fig. 2. Amplification of Fig. 1 emphasizing the recomposition regions and the different routes to chaos. The red and blue arrows are to guide the eyes.

and projections of the Lyapunov exponents on the space of the parameters. In Section 3 we present some final remarks and our conclusions. For interpretation of the references to color in the figures legend, the reader is referred to the web version of this article.

2. The model and results

The perturbed map is called as *squared sine logistic map* and it is given through the following mapping,

$$f(x_n) = x_{n+1} = rx_n(1 - x_n) + \mu \sin^2(2\pi x_n). \tag{1}$$

The square used in the periodic function ensures that the iterations remain in the interval $x \in [0, 1]$, when started with positive values of x, and μ is a second control parameter that alters the pattern of the original map and controls the intensity of the sinusoidal non-linear function. It is immediate to see that the value $x = 0$ is always a fixed point. However, the r-dependent fixed point of the logistic map can no longer be obtained analytically. After trying some values for μ, we kept it fixed in 0.2, even though any other value could be considered, and we have obtained the following results.

In Fig. 1, we vary r in [0, 4]. For each value of r we gave some initial values for x in order to search for possibly the coexistence of different attractors. Fig. 2 is an amplification of Fig. 1 in the range $r \in [2.6, 3.6]$. We observed that around $r \sim 2.73$, the second fixed point experiences a flip bifurcation, which bifurcates again for $r \sim 2.97$ and $r \sim 3.09$. In the range $r \in [3.09, 3.18]$ there is an attractor with period-8, however differently of the very well-known bifurcation cascade, for $r \sim 3.18$ occurs a recomposition of the attractor reducing its period to 4. From $r \sim 3.27$ up to $r \sim 3.36$ the attractor