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Q1 Synchrony-optimized networks of Kuramoto oscillators with inertia

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HIGHLIGHTS

- A rewiring algorithm for enhancing the synchronization capacity for networks of Kuramoto oscillators is proposed.
- The synchrony-optimized networks are robust and typically exhibit an anticipation of the synchronization onset.
- The main results are applied to the synchronization problem in large power grids.
- We study synthetic random networks and also a network with a topology approximating the (high voltage) power grid of Spain.

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ABSTRACT

We investigate synchronization in networks of Kuramoto oscillators with inertia. More specifically, we introduce a rewiring algorithm consisting basically in a *hill climb* scheme in which the edges of the network are swapped in order to enhance its synchronization capacity. We show that the synchrony-optimized networks generated by our algorithm have some interesting topological and dynamical properties. In particular, they typically exhibit an anticipation of the synchronization onset and are more robust against certain types of perturbations. We consider synthetic random networks and also a network with a topology based on an approximated model of the (high voltage) power grid of Spain, since networks of Kuramoto oscillators with inertia have been used recently as simplified models for power grids, for which synchronization is obviously a crucial issue. Despite the extreme simplifications adopted in these models, our results, among others recently obtained in the literature, may provide interesting principles to guide the future growth and development of real-world grids, specially in the case of a change of the current paradigm of centralized towards distributed generation power grids.

1. Introduction

Since Thomas Edison's Pearl Street Station in Manhattan started operating in 1882, the power grids have continued to grow and are today probably the largest machines ever built [1,2]. Their growth is still far away from being complete, since the pursuit of renewable sources of energy and new technologies drive the integration of different power grids into continental machines as, for instance, the paradigmatic case of Western Europe. (For an approximated description of the western European interconnected high voltage power grid, see Refs. [3,4].) The widespread use of alternating current

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creates the necessity of keeping the whole power grid synchronized, and a disruption in this synchronization may cause malfunctioning, leading to power outages with possible catastrophic proportions in real scenarios.

The phenomenon of synchronization has been studied for a long time [5–8] in different areas of knowledge. It is present in a myriad of situations, arising naturally in many areas of biology, physics, social sciences, etc. However, It has been only recently that a *complex system* approach has been devised to study the synchronization of power grids [9] (see also, for a recent review, [10]). Typically, a single power plant is a complicated machine, with a lot of tunable parameters necessary for its correct functioning. Although power grids can be, and surely are, analyzed and studied in all their finer details, taking into account hundreds of power plants, substations, transmission lines, and many other devices, the idea here is to focus on the complexity of the underlying network of connections [11,12] and its role on the overall synchronization process. In order to achieve such a goal, one treats the power plants as simple generators and the loads on the other side of transmission lines as passive machines. Energy balance in this context yields a set of equations known as the Kuramoto Model with inertia [8]. Recently, we have witnessed many works devoted to the analysis of power grids in this context of complex system as, for instance, the analysis of the European power grid [13], the effects of decentralization of energy production in the British power grid [14], the identification of parameters in individual vertices that turn the synchronous state more stable [15], the existence of Braess' paradox [16,17], the role of the topology [18–20] and assortative mixing [21] on the network synchronization and control, and a stability analysis of blackouts using basin-stability measures [22].

In this paper, we study the optimization, in order to favor synchronization, of networks of Kuramoto oscillators with inertia modeling power grids. More specifically, we adapt an algorithm previously proposed in Refs. [23,24] to optimize the synchronization capacity of a network built from usual Kuramoto oscillators to the case of oscillators with inertia. We study the main topological and dynamical properties of the synchrony-optimized networks generated by our algorithm and their robustness for edges (corresponding to transmission lines) removal and other perturbations which could mimic consumption peaks (or generation shortages) in real power grids. Our results show that the optimized networks tend to be more robust against the perturbations mimicking consumption peaks for a wider range of parameters when compared to the non-optimal networks, and no differences were detected with respect to edge removals.

The paper is organized as follows. In Section 2, we review briefly the power grid model based on Kuramoto oscillators with inertia [9] and discuss some of its properties with relevance to our analysis. Section 3 is devoted to the introduction of our optimization algorithm. In Section 4, we show the numerical results obtained for synthetic random networks and also for a network topologically based on an approximation of the Spanish high voltage power grid.

2. Kuramoto oscillators with inertia

For the sake of completeness, we will briefly review here the basic equations for the Kuramoto oscillators with inertia used in Ref. [9] for a simplified description of power grids. Our main goal is to derive some simple results concerning the synchronization of the underlying network which are important to our analysis. In this simplified description, a power grid is represented by an undirected network composed of $N = N_G + N_C$ vertices corresponding to two types of machines: N_G generators and N_C consumers (motors), which do not need to be equal in number necessarily. The power transmission lines correspond to the m edges connecting the vertices. The connectivity pattern is described by the usual symmetric adjacency matrix A, with elements a_{ij} such that $a_{ij} = 1$ if vertices i and j are connected, and $a_{ij} = 0$ otherwise.

Each individual element i of the network corresponds to a synchronous machine, generator or consumer, characterized by a power $\tilde{P}_i$, which is positive for generators and negative for the consumers. For each network vertex, simple energy balance implies that this power must be equal to the sum of three contributions: the rate of change of the machine kinetic energy

$$P_i^{\text{kin}} = I_i \ddot{\theta}_i \dot{\theta}_i \tag{1}$$

where I_i and θ_i stand for, respectively, the moment of inertia and the phase of the ith generator/consumer; the rate at which energy is dissipated through friction

$$P_i^{\text{diss}} = \gamma_i \dot{\theta}_i^2, \tag{2}$$

where γ_i is the dissipation constant associated with the machine at vertex i; and the total power transmitted to other vertices. In particular, the power transmitted from vertex i to j is given by

$$P_{ij}^{\text{trans}} = -P_{ij}^{\text{max}} \sin(\theta_j - \theta_i), \tag{3}$$

where P_{ij}^{max} represent the maximum power that can be transmitted along the transmission line connecting i and j vertices. Summing all terms, one has

$$\tilde{P}_i = I_i \ddot{\theta}_i \dot{\theta}_i + \gamma_i \dot{\theta}_i^2 - \sum_{j=1}^{N} P_{ij}^{\text{max}} \sin(\theta_j - \theta_i). \tag{4}$$

From now on, we restrict ourselves to the idealization often assumed for power grids in this context, namely that all elements in the grid have the same moment of inertia I and the same dissipation constant γ, and that all transmission lines have the