Physica A 441 (2016) 32–39

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Transitions induced by cross-correlated bounded noises and time delay in a genotype selection model

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HIGHLIGHTS

- Genotype selection model interplay with cross-correlated noises and time delay is investigated.
- Transition induced by cross-correlated bounded noises in the systems is observed.
- As time delay increases, probability distribution transfers from unimodal to bimodal.
- The system undergoes a new transition to four stable states for large correlation time.

ARTICLE INFO

Article history:
Received 5 April 2015
Received in revised form 12 June 2015
Available online 8 September 2015

Keywords:
Transitions
Time delay
Bounded noises
Genotype selection model

ABSTRACT

We present a numerical investigation of occurrence of transitions in a genotype selection model with time delay, where two bounded noises are cross-correlated. Stationary probability distribution (SPD) function is obtained. It is found that: the multiplicative bounded noise can facilitate the gene separation and it plays a constructive role in the genetic selection progress, while the additive bounded noise suppresses the gene separation. The strong correlation between noises gives a big chance to one type haploid out of the group. Besides, what is more interesting is that the correlation time τ can induce a new transitions (i.e., the curve of the SPD changes from unimodal to bimodal, and then to four peaks as the correlation time τ increases).

1. Introduction

The influence of noise on nonlinear systems has proved to be of considerable interest in recent years. Noise can play a constructive role in nonlinear systems, which is shown in many experimental facts. These researches include mean first-passage time analysis [1,2], steady-state analysis [3,4], stochastic transitions [5,6], stochastic resonance [7–10] and other effect. Stochastic transitions, as one of the most interesting phenomena induced by noise in random dynamical systems, has become a rapid and hot topic for physics, chemistry, economics and biology [1–10], and is generally characterized with a qualitative change of the SPD, e.g., a transition from unimodal to bimodal distribution or vice versa. In Ref. [11], the authors explored a molecular regulatory systems with three different types noise sources. Result shows that the type of noise determines the strongly attracting steady state or stochastic attractor. In Ref. [12], the authors demonstrated the issue of how intrinsic noise modulates stochastic switching rate using two scenario. Since the roles and benefits of stochastic phenomena in natural systems are starting to be elucidated, it becomes relevant to characterize the features of such stochastic phenomena in terms of the driving fluctuations.

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http://dx.doi.org/10.1016/j.physa.2015.08.036
0378-4371/

Stochastic fluctuations are ubiquitous in any real natural dynamical system. In particular, living organisms are subject to fluctuations (or noise) of distinct origins. At a cellular level, it is a well-known fact that biochemical reactions inside a cell are stochastic events and present inherent randomness. These fluctuations can have disturbing or ordering roles. For instance, noise may act as a trigger for phenotypic variability since the exploration of the phase space through different types of dynamics [13–15]. However, in most of the previous works, the fluctuation was assumed to be Gaussian process, which is unbounded and may reach large values [16,17]. This fact contradicts the very nature of a real physical quantity which is always bounded [18,19]. The conclusions that have been built on Gaussianity assumptions should be reconsidered and a more suitable model for the bounded interference is required. Many experiments in physics, biology, engineering [20], neural networks [21,22], as well as sensory systems [23–25], have proved that the noise source in these systems is general bounded noise. Based on such a consideration, M.F. Dimentberg [26] and W.V. Wedig [27] put forward a versatile model for bounded random processes, by using a sinusoidal function with a constant amplitude, a constant average frequency and a random phase varying as a Wiener process. Meanwhile, another type of bounded noise is well-known as sine-Wiener (SW) noise [19], which induces transitions in different models [19,28,29]. Nevertheless, the investigations of dynamical systems with bounded noise are complicated [19,20,30], and the research in this field is pretty rare so far. In addition, time delay is unavoidable in nature, and the inclusion of time delay term is more reasonable in realistic systems. From the point of physics, time delay, which plays an important role in the dynamical propertied of stochastic systems [31,32], usually arises from the finite transport time of matter, energy and information. However, most studies neglect the possible effect caused by time delay. This is mainly because of the difficulty in analytic methods of dealing the non-Markovian nature of the delayed stochastic systems [33,34].

Recently, the investigations of dynamical actions in genetic selection process have drawn widespread concern due to its wide potential application, such as gene separation, biological evolution, bio-pharmacy and so on. It is necessary to understand selection mechanism and develop relevant strategies to satisfy our human demands. In literatures [35,36], the genotype selection model only driven by multiplicative noise has been studied. It indicates that white multiplicative noise can induce transitions. Zhang and Cao researched the effect of correlated noise on genetic model [37]. The effect of Ornstein–Uhlenbeck noise on genetic has been studied in Refs. [36,38,39], and the effect of a non-Gaussian noise on that model have been investigated in Ref. [40]. F. Castro et al., for first time in Ref. [41], found the reentrance behavior in the genic model, which is subjected to a multiplicative color noise source. All these researches reveal that non-extensive parameter can induce transitions which contributes to gene selection. Environmental fluctuation plays a crucial role in gene selection. Meanwhile, the effect of time delay on a genotype selection model driven by white multiplicative noise has been investigated in Ref. [42], which shows that time delay is an important parameter affecting transitions, and it is helpful to select one type of genes from another type of mixing genes. However, all these works neglected the bounded nature of noise, which is more reasonable for the real physical quantity. Therefore, it is worthy to introduce suitable bounded noise into a genotype selection progress. Moreover, the synergistic effect of bounded noise with time delay on the genetic selection progress should be further researched.

In this paper, we aim to explore random dynamical behaviors of a genotype selection model with time delay in the present of cross-correlated sine-Wiener (CCSW) noises, which is expected to be a more powerful tool to demonstrate the interactions between the genotype selection system and its surrounding. In particular, we will concern the stochastic transitions induced by CCSW noises and time delay, and its biological interpretations. The paper is organized as follows. Section 2 is devoted to the model with time delay, driven by cross-correlated sine-Wiener noises. In Section 3, based on the SPD of the model, transitions induced by noises and time delay are discussed. In Section 4, we end the paper with concluding remarks.

2. Model and CCSW noises

2.1. Model

We select a haploid group as our object and suppose that each haploid may have gene A or B. The simple genotype selection model can be described as one-variable differential equation [35,36,43],

$$\dot{x} = \beta - x + \mu x(1 - x), \tag{1}$$

where x is the ratio of the number of gene A to the total number, so $x \in [0, 1]$. β stand for the mutation rate of gene B, $\beta \in (0, 1)$. μ denote gene selection factor, $\mu = 1$ means that the selection is completely propitious to gene A haploid, $\mu = -1$ means that the selection is completely propitious to gene B haploid. No matter how the parameter β or μ changes, this equation has only one equilibrium point in the corresponding interval, i.e.,

$$x_0 = \frac{\mu - 1 + \sqrt{(\mu - 1)^2 + 4\beta\mu}}{2\mu}.$$

This result indicates that there is no phase transition during the process of gene selection for the deterministic Eq. (1).

Notice that the second term in Eq. (1) represents gene transformation and regeneration. There should be a reaction time of gene number to their surrounding environment fluctuation. Supposing a constant delay time τ_α, x item should become $x(t - \tau_\alpha)$.