Physica A 441 (2016) 40–50

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Thermodynamics of inequalities: From precariousness to economic stratification

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HIGHLIGHTS

- Economic stratification is measured by the entropy of the wealth distribution.
- The fluctuation theorem implies a "second-law inequality" for stratification.
- Precariousness is the thermodynamic force conjugate to upward economic mobility.
- Precariousness and upward economic mobility together drive stratification up.
- We estimate the relaxation time of the wealth distribution in a diffusion model.

ARTICLE INFO

Article history:
Received 7 October 2014
Received in revised form 8 May 2015
Available online 9 September 2015

Keywords:
Econophysics
Wealth inequality
Second law of thermodynamics
Fluctuation theorem
Markov process
Entropy

ABSTRACT

Growing economic inequalities are observed in several countries throughout the world. Following Pareto, the power-law structure of these inequalities has been the subject of much theoretical and empirical work. But their *nonequilibrium dynamics*, e.g. after a policy change, remains incompletely understood. Here we introduce a thermodynamical theory of inequalities based on the analogy between economic stratification and statistical entropy. Within this framework we identify the combination of *upward mobility* with *precariousness* as a fundamental driver of inequality. We formalize this statement by a "second-law" inequality displaying upward mobility and precariousness as thermodynamic conjugate variables. We estimate the time scale for the "relaxation" of the wealth distribution after a sudden change of the after-tax return on capital. Our method can be generalized to gain insight the dynamics of inequalities in any Markovian model of socioeconomic interactions.

1. Introduction

All known human societies[1] have displayed some level of economic inequality [2]. Yet this global imbalance is reaching alarming levels in the contemporary world: as of 2013, the 400 richest Americans have more wealth than the bottom half of all Americans combined. Indeed recent comprehensive research [3] has showed that, while they have not reached the highs of the pre-1929 period, wealth inequalities in developed countries have steadily increased in the past decades. Understanding the origins and implications of these inequalities is an outstanding problem for economics, but also for society as a whole.

On the theory side, a well-established approach to this problem – pursued independently by economists [4], mathematicians [5,6], sociologists [7] and physicists [8–10] – consists in studying the equilibrium wealth distribution in stochastic

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[1] Dating back to paleolithic hunter–gatherers [1].

http://dx.doi.org/10.1016/j.physa.2015.09.001
0378-4371/

models of individual (or household) income. Under general assumptions, one shows that additive income lead to exponential distributions, while multiplicative capital returns yield Pareto-like power law distributions [11,12]. These results are consistent with empirical data, both contemporary [13] and historical [14], which reveal a two-class structure with an exponential range at low wealth (where investment is negligible) and a power-law tail at high capital (where income is dominated by investment returns). Econophysicists have pointed the striking similarity between this pattern and the Boltzmann–Gibbs distribution of statistical mechanics [10]. Indeed both have the same "entropic" structure—there are many more ways to distribute a conserved quantity (be it wealth or energy) unequally than equally. The econophysics approach to economic inequalities is discussed in Refs. [15–18].

One much discussed consequence of such marked economic inequalities is the emergence of a super-elite class, the so-called "top 1%" [19], with disproportionate social, economical and political influence. But they also have more global effects, one of which is increased *stratification* [20]—the growth of the number of economically distinct "classes" in society. Indeed, as we will see below, "maximum entropy" wealth distributions are precisely those with the greatest stratification under global constraints on the mean wealth. This intriguing analogy between entropy and stratification points to a connection between the dynamics of inequalities and the physics of dissipation in thermal systems, extending beyond the limits of equilibrium statistical mechanics (to which it has been restricted so far).

In this paper we introduce a general framework, inspired from stochastic thermodynamics [21], to account for the dynamical origin of social inequalities. At its foundation is a general property of Markov processes known as the *fluctuation theorem*[2] (Appendix A). As we shall see, the great strength of this theorem lies in its *explanatory power*: given an entropy-increasing stochastic process, the fluctuation theorem elucidates the *mechanism* driving entropy production. In the context of social inequalities, where entropy quantifies inequality, we find that, over and above the multiplicative effect of capital return, *precarious social mobility* acts as a universal inequality-generating mechanism.

2. Results

2.1. Stratification

We begin by formalizing our notion of stratification. Let $w \in [w_{\min}, w_{\max}]$ denote the wealth of an individual (or household) in the economy. The *wealth distribution* $p_t(w)$ is the probability density function (PDF) at time t for the wealth variable w, i.e. $p_t(w)\mathrm{d}w$ gives the probability of finding an agent with wealth at time t between w and $w + \mathrm{d}w$, or the fraction of population whose wealth is between w and $w + \mathrm{d}w$ at that time.

Given δw a reference wealth unit, we call economic *stratum* a segment of the population with wealth in the range $[w_i, w_{i+1}]$ where $w_i = w_{\min} + b^i \delta w$ for some conventional number $b > 1$. For instance, we could take $\delta w = \$1$ and $b = 10^3$, in which case the words "millionaire" and "billionaire" would correspond to the adjacent strata $i = 3$ and $i = 4$.

Next we define the *stratification* S_t of the population at time t by

$$S_t \equiv -\int_{w_{\min}}^{w_{\min}} p_t(w) \log[p_t(w)\delta w]\, \mathrm{d}w, \tag{1}$$

where log is the base b logarithm. (Mathematically, S_t is the "differential entropy" of the wealth distribution $p_t(w)$.) Stratification is maximized by the uniform distribution on the interval $[w_{\min}, w_{\max}]$, in which case it simply measures the *number of strata* in the population. This feature is to be contrasted with the Gini index commonly used in the social sciences to measure economic inequalities:

$$G_t \equiv 1 - \frac{1}{\langle w \rangle_t} \int_{w_{\min}}^{w_{\min}} \left[1 - \int_{w_{\min}}^{w} p_t(w')\, \mathrm{d}w'\right]^2 \mathrm{d}w \tag{2}$$

where $\langle w \rangle_t$ is the mean of the distribution $p_t(w)$. Indeed, G_t is maximized not by uniform wealth distributions, but by the (highly unrealistic) "state of extreme inequality" in which one agent has all wealth, and all $N - 1$ other agents have nothing: $p(w) = (1 - N^{-1})\delta(w - w_{\min}) + N^{-1}\delta(w - w_{\max})$. The fact that the Gini index is maximized by such a singular distribution makes it rather unnatural in the context of large populations with smooth, unimodal distributions. This being said, in many cases of interest the Gini index turns out to be an increasing function of stratification, as illustrated in Fig. 1.

It is remarkable that both the Boltzmann (exponential) and Pareto (power-law) distributions,[3] which have been argued to describe the empirical wealth distributions in the lower and higher quantiles respectively, arise as maximum stratification distributions. Indeed, the former corresponds to the maximum of S under the constraint $\langle w \rangle = 1/\beta$, while the latter corresponds to the maximum of S under the constraint $\langle \log(w/w_{\min}) \rangle = 1/\alpha$. (One can check that S is a monotonically decreasing function of β and α respectively.) In other words, the lower (resp. higher) quantiles of society appear to be

[2] Originally discovered in the context of non-equilibrium statistical mechanics, this result has been successfully applied to models of evolutionary dynamics [22] and of biopoiesis [23]. More non-physics applications will likely come in the near future.

[3] The Boltzmann distribution at "inverse temperature" β is $p_B(w) = \beta e^{-\beta w}$, with stratification $1 - \log(\beta \delta w)$. The Pareto distribution within minimum wealth $w_{\min}$ and Pareto index α is $p_P(w) = \alpha w_{\min}^{\alpha}/w^{\alpha+1}$, with stratification $1 + 1/\alpha + \log(w_{\min}/\alpha \delta w)$.