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Holographic considerations on non-gaussian statistics and gravothermal catastrophe

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HIGHLIGHTS

- Comparison of Tsallis and Kaniadakis statistical formalisms.
- Use of the Verlinde's holographic approach.
- Gravothermal catastrophe phenomena analyzed.

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ABSTRACT

In this paper we have derived the equipartition law of energy using Tsallis formalism and the Kaniadakis power law statistics in order to obtain a modified gravitational constant. We have applied this result in the gravothermal collapse phenomenon. We have discussed the equivalence between Tsallis and the Kaniadakis statistics in the context of Verlinde's entropic formalism. In the same way we have analyzed the negative heat capacities in the light of gravothermal catastrophe. The relative deviations of the modified gravitational constants are derived.

1. Introduction

The mechanism of gravothermal instability, discovered by Antonov [1–4] is an important phenomena in gravitational thermodynamics. It has been very helpful for an extensive research concerning statistical mechanics of long range interactions systems in several fields in physics [5]. This connection with thermodynamics and statistical mechanics has motivated us to investigate statistically the gravothermal catastrophe.

At the same time, there are theoretical evidences that the understanding of gravity has been greatly benefited from a possible connection with thermodynamics. Pioneering works of Bekenstein [6] and Hawking [7] have described this issue.

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For example, quantities as area and mass of black-holes are associated with entropy and temperature respectively. Working on this subject, Jacobson [8] interpreted Einstein field equations as a thermodynamic identity. Padmanabhan [9] gave an interpretation of gravity as an equipartition theorem.

Recently, Verlinde [10] brought a heuristic derivation of gravity, both Newtonian and relativistic, at least for static spacetime. The equipartition law of energy has also played an important role. On the other hand, one can ask: what is the contribution of gravitational models in thermodynamics theories?

The concept introduced by Verlinde is analogous to Jacobson's [8], who proposed a thermodynamic derivation of Einstein's equations in Rindler's space. The result has shown that the gravitation law derived by Newton can be interpreted as an entropic force originated by perturbations in the information "manifold" caused by the motion of a massive body when it moves away from the holographic screen. Verlinde used this idea together with the Unruh result [11] and he obtained Newton's second law. The entropic force together with the holographic principle and the equipartition law of energy resulted in Newton's law of gravitation. Besides, Verlinde's ideas have been used in cosmology [12].

Currently, two of the most investigated extensions of the usual Boltzmann–Gibbs (BG) theory are Tsallis thermostatistics theory [13–15] (TT) and Kaniadakis power law statistics [16–18] (KS). The former case, TT, initially considers the entropy formula as a nonextensive (NE) quantity where there is a parameter q that measures the so-called degree of nonextensivity. This formalism has been successfully applied in many physical models. An important feature is that when $q \rightarrow 1$ we recover the usual BG theory. On the other hand, the so-called KS naturally emerges from the context of special relativity and in the kinetic interaction principle. This formalism has also been successfully applied in many physical models. There is a parameter κ in KS that in the limit $\kappa \rightarrow 0$ the BG theory is also recovered.

This paper has two parts: in the first one, the self-gravitating system [19] (and references therein) is discussed in the context of TT and KS formalisms. We found that for both formalisms the dependence of the specific heat, C_V, on the nonextensive parameter q and on the κ parameter gives rise to a negative branch, a result that features the gravothermal collapse. In the second part, we have used TT and KS formalisms in the Verlinde one [10]. As a result, the equipartition theorem is derived in the framework of KS leading to a modified gravitational constant.

This paper is organized as follows: in Section 2 the main steps of Tsallis, Kaniadakis and Verlinde's formalisms are reviewed. In Section 3 a modified gravitational constant is obtained in the light of Kaniadakis' formalism. In Section 4, in an holographic background, we have obtained relations between q and κ. In Section 5 the gravothermal collapse is studied in the light of TT and KS formalisms. In Section 6 the relative errors of the gravitational constants are calculated. The conclusions and perspectives are presented in the last section.

2. Non-gaussian statistics and holographic entropy

The study of entropy has been an interesting task through recent years thanks to the fact that it can be understood as a measure of information loss concerning the microscopic degrees of freedom of a physical system, when describing it in terms of macroscopic variables. Appearing in different scenarios, we can conclude that entropy can be considered as a consequence of the gravitational framework [6,7]. These issues motivated some of us to consider other alternatives to the standard BG theory in order to work with Verlinde's ideas together with other subjects [12].

The objective of this section is to provide the reader with the main tools that will be used in the following sections. Although both formalisms are well known in the literature, these brief reviews can emphasize precisely that there is a connection between both ideas which was established recently [20].

2.1. Tsallis' formalism

Tsallis [13] has proposed an important extension of BG statistical theory and curiously, in a technical terminology, this model is also currently referred to as nonextensive statistical mechanics. TT formalism defines a nonadditive entropy given by

$$S_q = k_B \frac{1-\sum_{i=1}^{W} p_i^q}{q-1} \quad \left(\sum_{i=1}^{W} p_i = 1\right), \tag{2.1}$$

where p_i is the probability of the system to be in a microstate, W is the total number of configurations and q, known in the current literature as Tsallis parameter or NE parameter, is a real parameter which quantifies the degree of nonextensivity. The definition of entropy in TT formalism possesses the usual properties of positivity, equiprobability, concavity and irreversibility and motivated the study of multifractals systems. It is important to stress that Tsallis formalism contains the BG statistics as a particular case in the limit $q \rightarrow 1$ where the usual additivity of entropy is recovered. Plastino and Lima [21] have derived a NE equipartition law of energy. It has been shown that the kinetic foundations of Tsallis' NE statistics lead to a velocity distribution for free particles given by [22]

$$f_q(v) = B_q \left[1-(1-q)\frac{mv^2}{2k_BT}\right]^{1/1-q}, \tag{2.2}$$