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# Preferential spreading on scale-free networks

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ABSTRACT

Based on a classical contact model, the spreading dynamics on scale-free networks is investigated by taking into account exponential preferentiality in both sending out and accepting processes. In order to reveal the macroscopic and microscopic dynamic features of the networks, the total infection density $\rho$ and the infection distribution $\rho(k)$, respectively, are discussed under various preferential characters. It is found that no matter what preferential accepting strategy is taken, priority given to small degree nodes in the sending out process increases the total infection density $\rho$. To generate maximum total infection density, the unbiased preferential accepting strategy is the most effective one. On a microscopic scale, a small growth of the infection distribution $\rho(k)$ for small degree classes can lead to a considerable increase of $\rho$. Our investigation, from both macroscopic and microscopic perspectives, consistently reveals the important role the small degree nodes play in the spreading dynamics on scale-free networks.



## 1. Introduction

Preferentiality exists widely among social networks. For instance, advertisers prefer to use celebrities as spokespersons for their products in order to popularize their brand and enlarge their sale market. Therefore preferentiality does indeed influence the spreading process, and should be taken into account in the spreading dynamics on networks. As a typical example, epidemic spreading [1–12] has been studied extensively to understand the spreading dynamics on real networks since the discoveries of small-world [13] and scale-free [14] networks [15,16]. These investigations were mainly based on susceptible-infected (SI) [7,8], susceptible-infected-susceptible (SIS) [10,11], and susceptible-infected-removed (SIR) [3,12] models. Studies on scale-free networks showed that topological heterogeneity leads to exceptional features such as the absence of an epidemic threshold [11]. Besides the essential features of threshold [9–11], further applications such as how to restrain the spreading of computer viruses on the Internet by immunization are also highly investigated. Targeted immunization [6] and acquaintance immunization [17] have been demonstrated as two effective immunization strategies. On the other hand, close attention is also paid to issues such as how to boost spreading efficiency by adopting some special spreading strategies so that it can be applied to systems such as broadcasting. Zhou et al. [7] introduced a fast spreading strategy, i.e. SI model, and found that the preference on small degree nodes is more efficient than that on large degree nodes. Yang et al. [18] investigated the spreading dynamics on the basis of a contact process model, and revealed that frequently choosing small degree nodes leads to a large infection spreading. They studied the infection density of the whole network macroscopically, given a power law $k^{\beta}$ of preferentiality governing the spreading process [7,18,19]. While in some systems, the preferentiality may depend more strongly on node degree. To explore the dynamic process in these systems, we study the spreading process on scale-free networks governed by an exponential law of preferential strategy $e^{\beta k}$ describing the stronger dependence of preferentiality on nodes of different degrees. The evolution of infection density for nodes with different degrees under various preferential characters is presented and discussed. The important role small degree nodes play during the spreading process is confirmed.

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## 2. Model and analysis

Our model, which is parallel concerning its dynamic simulation, is based on the classical contact process (CP) [20,21] on a scale-free network, in which each node is either infected or susceptible to infection. Initially, a fraction $\rho_0$ of nodes on the network are infected and each infected node is marked by one particle on it. In each evolution step, the particle will perish with probability $p$, or survive with probability $1 - p$ and generate a new particle in order to infect one of its neighboring nodes. One complete contact process is composed of a sending out process and an accepting process [18,19], with probabilities that are both exponentially degree-dependent. Suppose node $i$ is occupied by a surviving particle at the time step $t$, then, a newly generated particle will be passed to one of its neighboring nodes $j$ of degree $k_j$ with probability

$$\phi_{\text{out}} = \frac{e^{\beta k_j}}{\sum\limits_{l \in V_i} e^{\beta k_l}}. \tag{1}$$

Here $V_i$ denotes all the neighbors of node $i$, and the sending out parameter $\beta$ denotes the preferential character. That is, if $\beta > 0$, a neighbor with large degree is more likely to be chosen as the target; while if $\beta < 0$, that with small degree is more likely to be infected; $\beta = 0$ corresponds to the unbiased preferential strategy ($\phi_{\text{out}} = 1/V_i$). Besides the sending out process, the accepting process is also governed by preferentiality [18]. Given $\alpha$ as the accepting parameter, we assume that the target node $j$ will accept the new particle from node $i$ following a probability

$$\phi_{\text{in}} = \frac{e^{\alpha k_i}}{\max(e^{\alpha k_m} \mid m \in V_j)}. \tag{2}$$

Here, $V_j$ denotes all the neighbors of node $j$. If $\alpha > 0$, a target prefers to accept one new particle from a neighbor with large degree; if $\alpha < 0$, a new particle from a small degree neighbor is more welcome; while if $\alpha = 0$, the preference is unbiased ($\phi_{\text{in}} = 1$). Moreover, it is clear from Eq. (2) that an unbiased accepting strategy ($\alpha = 0$) leads to a maximum total infection density, as for $\alpha = 0$, $\phi_{\text{in}} = 1$ for all new particles from neighbors of various degrees $k$; while for $\alpha \neq 0$, $\phi_{\text{in}} < 1$ for all new particles expect those from neighbors of the maximum degree $k_{\max}$ when $\alpha > 0$ and those from neighbors of the minimum degree $k_{\min}$ when $\alpha < 0$, indicating that exponentially biased accepting, as given by Eq. (2), plays a negative role in the spreading process. Therefore, as a complete spreading process, a newly generated particle will move from the infected node $i$ to the target node $j$ with probability $\phi = \phi_{\text{out}} \cdot \phi_{\text{in}}$. Each infected or susceptible node can act as a target, and can be selected by several infected neighbors in one evolution step. However, within each step, new particles accepted by an infected target are wasted, and several new particles accepted by a susceptible target play the same role as one new particle.

As the spreading takes place through a contact between infected nodes and their neighbors, the neighboring degree distribution, not just the degree distribution of the whole network, plays an important role during the contact process. By introducing the conditional probability $P(k'|k)$ that a node of degree $k$ is connected to a node of degree $k'$ [22], the total number of links between nodes of degree $k_A$ and nodes of degree $k_B$ can be written as $L(k_A, k_B) = NP(k_A)k_AP(k_B|k_A)$. For an uncorrelated network, such as the Barabási–Albert (BA) model [14], where $P(k'|k) = k'P(k')/\langle k\rangle$ [23], one obtains $L(k_A, k_B) = NP(k_A)k_Ak_BP(k_B)/\langle k\rangle$. As the amount of nodes of degree $k_A$ takes the form $N(k_A) = NP(k_A)$, each node of degree $k_A$ averagely has

$$n(k_A, k_B) = \frac{L(k_A, k_B)}{N(k_A)} = \frac{1}{\langle k\rangle} k_Ak_BP(k_B) \tag{3}$$

neighbors of degree $k_B$. Since the degree distribution of the BA model takes the power law form $P(k) \sim k^{-\gamma}$ with $\gamma = 3$ [14], the neighboring degree distribution for a given degree $k_A$ also scales as a power law $n(k_A, k_B) \sim {k_B}^{-2}$, indicating that a node, whatever its degree is, averagely has much more neighbors of small degrees than those of large degrees.

On the other hand, preferential characters are key factors impacting the spreading dynamics. Fig. 1 shows the average sending out priority varying with the sending out preference. The average sending out priority $\sigma_{\text{out}}(k_A, k_B)$ from an infected node of degree $k_A$ to a neighboring node of degree $k_B$ is expressed as

$$\sigma_{\text{out}}(k_A, k_B) = \frac{e^{\beta k_B}}{\sum\limits_{k_B} \frac{L(k_A, k_B)}{N(k_A)} e^{\beta k_B}}, \tag{4}$$

where $L(k_A, k_B)/N(k_A)$ denotes the average number of links from a node of degree $k_A$ to that of degree $k_B$. As

$$\sum_{k_B} \frac{L(k_A, k_B)}{N(k_A)} = k_A, \tag{5}$$

when the sending out process is unbiased, i.e. $\beta = 0$, $\sigma_{\text{out}}(k_A, k_B) = 1/k_A$, indicating that $\sigma_{\text{out}}(k_A, k_B)$ is equal for all the neighbors (Fig. 1(a)). When $\beta > 0$, preference on large degree neighbors leads to a monotonous increase of $\sigma_{\text{out}}(k_A, k_B)$ with $k_B$ for a given degree $k_A$ (Fig. 1(b)). When $\beta < 0$, neighbors with small degrees are more likely to be chosen as the target, and $\sigma_{\text{out}}(k_A, k_B)$ decreases monotonously with $k_B$ for a given $k_A$ (Fig. 1(c)). Strong preferentiality, represented by a large value of $\beta$, leads to intense divergence in the sending out priority among neighbors with different degrees.