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The scaling properties of stock markets based on modified multiscale multifractal detrended fluctuation analysis

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HIGHLIGHTS

- Investigate the scaling structures in Chinese stock markets and US stock markets by performing modified MMA method.
- Focus on the distribution histograms of Hurst surface.
- Allow the assessment of more universal and subtle scaling characteristics.

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ABSTRACT

Here we propose the new method DH-MMA, based on multiscale multifractal detrended fluctuation analysis(MMA), to investigate the scaling properties in stock markets. It is demonstrated that our approach can provide a more stable and faithful description of the scaling properties in comprehensive range rather than fixing the window length and slide length. It allows the assessment of more universal and subtle scaling characteristics. We illustrate DH-MMA by selecting power-law artificial data sets and six stock markets from US and China. The US stocks exhibit very strong multifractality for positive values of q, however, the Chinese stocks show stronger multifractality for negative q than positive q. In general, the US stock markets show similar behaviors, but Chinese stock markets display distinguishing characteristics.

1. Introduction

Financial markets are considered to be extremely complex dynamical systems with a large number of interacting units such as traders, banks, mutual funds and assets. The dynamics of financial markets is difficult to understand due to the complex structure. It is widely accepted that financial markets illustrate strong signs of complexity, volatility clustering, power-law and multifractality [1–14]. Multifractality is a well known characteristic of complex dynamics such as DNA sequences, heart beat rate, weather records, financial markets, sunspot numbers [15–19]. Multifractal detrended fluctuation analysis (MF-DFA) method [20–22] is a universal tool to investigate multifractality, which is a multifractal generalization of the detrended fluctuation analysis (DFA) method [15].

Based on the former studies, the fluctuation scaling of monofractal time series may be described by a single exponent, and in most cases, the scaling of multifractal time series is interpreted by two coefficients. However, it is not adequate to describe the dynamical behaviors of time series by using one single or two scaling exponents. In order to avoid errors

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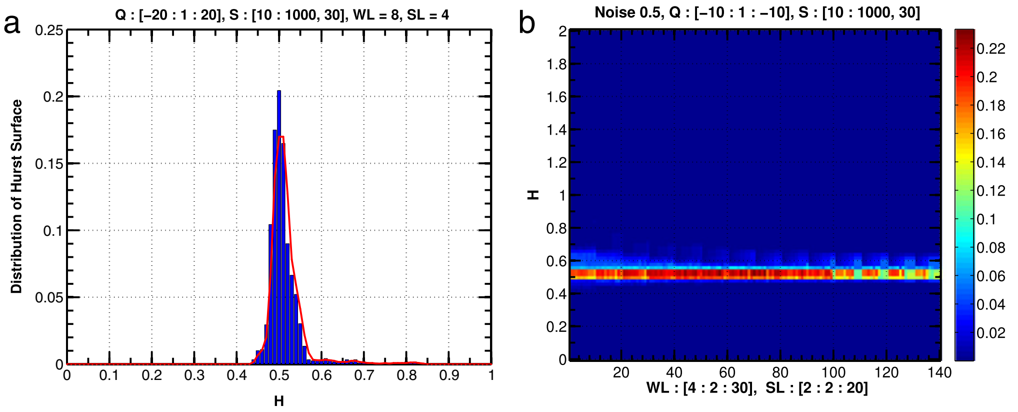

Fig. 1. (a) The distribution of Hurst surface $h(q, s)$ versus scaling exponents H for random series. The $h(q, s)$ is evaluated with parameters $Q = [-20 : 1 : 20]$, $S = [10 : 1000, 30]$, $WL = 8$ and $SL = 4$. The red line is D_{h_1}. (b) Scaling exponents H as a function of different cases of WL and SL. Different colors represent values of D_{h_i}, $i = 1, 2, \dots, 140$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

due to improperly predefined scaling ranges, and to obtain all information among the entire time scales, J. Gierałtowski et al. [24] generated a new method called multiscale multifractal detrended fluctuation analysis (MMA) for solving the upper problem. This new technology allows us to investigate not only the multifractal properties but also dependence of these properties on the time scale. MMA has successfully been applied to diverse fields such as heart rate dynamics [25], economics time series [26] and traffic dynamics [27]. In this paper, we decide to go further and discuss more details about the scaling behaviors based on MMA method. The objective of the present work is the development of methodology and the evaluation of multifractal scaling behaviors of stock markets by modified MMA technique. MMA results are presented as Hurst surface, thus we focus on the distribution histogram of Hurst surface to quantify the multifractal scaling properties of stock markets. Hence, our results are more stable and robust. Such information would be very useful for investors to correctly assess the risk in investments and also for policy makers to make the appropriate decisions. In the future we may also apply some machine learning techniques for the purpose of stock market data analysis, such as techniques used in Refs. [28,29].

The organization of this paper is as follows. Section 2 presents methods employed in study. Section 3 describes the data sets used in our work. Section 4 is devoted to show the results by employing our modified approach. Section 5 gives the conclusions.

2. Methodology

2.1. MF-DFA method

The multifractal detrended fluctuation analysis (MF-DFA) method was developed by Kantelhardt et al. [20] for the multifractal characterization of non-stationary time series. MF-DFA is a generalization of the Detrended Fluctuation Analysis (DFA) method. MF-DFA method can be described as follows. Let us suppose that x_t is a series of length N, and this series is of compact support, i.e. $x_t = 0$ for an insignificant fraction of the values only. The corresponding profile $Y(i)$ is computed by integration as

$$Y(i) = \sum_{t=1}^{i} (x_t - \langle x \rangle), \quad i = 1, 2, \dots, N. \tag{1}$$

Cut the profile $Y(i)$ into $N_s \equiv [N/s]$ non-overlapping segments of equal length s. Since the record length N need not be a multiple of the considered time scale s, a short part at the end of the profile will remain in most cases. In order not to disregard this part of the record, the same procedure is repeated starting from the other end of the record. Now the local trend $y_\nu(i)$ for each $2N_s$ segments is determined by the least square fit and then the variance is calculated using

$$F^2(s, \nu) = \frac{1}{s} \sum_{i=1}^{s} \{Y[(\nu - 1)s + i] - y_\nu(i)\}^2 \tag{2}$$

for $\nu = 1, 2, \dots, N_s$ and

$$F^2(s, \nu) = \frac{1}{s} \sum_{i=1}^{s} \{Y[N - (\nu - N_s)s + i] - y_\nu(i)\}^2 \tag{3}$$