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Lattice bosons in a quasi-disordered environment: The effects of a superlattice potential on single particle and many particle properties

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HIGHLIGHTS

- We study the effects of superlattice and Aubry–André potentials on 1d lattice bosons.
- We find a re-entrant localization–delocalization transition.
- We also find the development of multiple mobility edges.
- The superlattice potential significantly affects the condensate depletion.

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ABSTRACT

In this paper we present a theoretical investigation of the effect of a superlattice potential on some properties of non-interacting bosons in one dimensional lattices with Aubry–André disorder potential. In the first part, we investigate the single particle localization properties. We find a re-entrant localization–delocalization transition and the development of multiple mobility edges for a range of superlattice potential strengths. In the second part, we study the Bose–Einstein condensation with an additional harmonic trapping potential. We find that an increase in the superlattice potential leads to an increase in the depletion of the condensate in the low temperature limit.

1. Introduction

Studies of localization of the de Broglie waves in disordered and quasi-disordered environments have received renewed attention in the recent years primarily due to the direct observation of Anderson localization [1] of these matter waves in cold atom experiments [2–4]. While random disorder localizes all states in one dimension [5], a deterministic disorder distribution like the one in the Aubry–André model (AA) [6] leaves all states extended if the disorder strength remains below a critical value. The AA model [6] (or the Harper model [7]) has already been experimentally realized and a detailed study of its localization properties conducted [8,9]. In the AA model, since all the states become localized beyond a critical disorder strength, there is no mobility edge. Recent theoretical studies [10–15] have discovered the development of mobility edges in several extended AA models. Continuing along this direction of research, in this paper we consider the effects of a superlattice potential on the single particle and collective properties of non-interacting bosons in a one dimensional AA model. Among

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other results, we find a re-entrant single particle localization–delocalization transition with increasing superlattice potential strength.

The rest of this paper is organized as follows. The studies of the single particle localization properties are presented in Section 2. The Section 3 deals with the effects of the superlattice potential on the Bose–Einstein condensation. The conclusions are given in Section 4.

2. The changes in the localization properties due to the superlattice potential

In this section, we consider the effect of a superlattice potential on the single particle localization properties. Consider a lattice boson moving in a one-dimensional lattice with AA disorder and an additional superlattice potential. For this system, the Hamiltonian is:

$$H = -t \sum_{\langle ij \rangle} \left(c_i^\dagger c_j + c_j^\dagger c_i \right) + \sum_i \left[\lambda \cos(2\pi q i) + (-1)^i V \right] c_i^\dagger c_i, \tag{1}$$

where $t\ (> 0)$ is the energy gain when a boson hops from site i to its nearest neighbor (NN) site j, $c_i^\dagger$ is a creation operator of a boson at site i, λ the strength of the AA potential, $q = (\sqrt{5} + 1)/2$ is the incommensurability parameter, and V is the strength of the superlattice potential. Here t, λ, and V have energy units. All the energies in this paper are measured in units of t and all the lengths are measured in units of the lattice parameter. When $V = 0$, one obtains the AA model and energy independent localization of all the states for $\lambda > 2$, and thus there is no mobility edge. To study the effects of finite V on the localization properties, we first write H in the single particle site basis and then numerically diagonalize it to obtain its eigen-energies and eigen-functions. All our results presented in this paper are for a lattice of 610 sites. We monitor the localization of a given eigen-state with amplitude a_i at site i

$$|\psi\rangle = \sum_i a_i \, |i\rangle \tag{2}$$

by calculating its Inverse Participation Ratio (IPR) defined by

$$IPR = \frac{\sum_i p_i^2}{\left(\sum_i p_i \right)^2}, \tag{3}$$

where,

$$p_i = |\langle i|\psi\rangle|^2 = |a_i|^2. \tag{4}$$

In Fig. 1 we have displayed the IPR values of all the 610 states for various values (<2) of λ. For these values of λ, all the states are extended in the absence of V. We find that increasing V drives most states to localized. We also notice that the band edge states are more susceptible to localization and become localized for smaller values of V. When we increase λ another qualitatively different behavior is found. In the results shown for $\lambda = 1.3$ and 1.5, we notice that some states exhibit a re-entrant localization–delocalization transition. We do not find any re-entrant behavior with increasing λ for a fixed V as shown in Fig. 2. A consequence of the localization–delocalization transition is the enhancement of the entanglement with increasing V for the concerned states in relevant regions, as shown in Fig. 3. The entanglement is measured by the Shannon entropy [16] defined by

$$S = -\sum_i p_i \log_2 \, p_i. \tag{5}$$

The variation of the single particle spectrum with V for $\lambda = 1.5$ is shown in Fig. 4. It is clear that for several values of V the system develops *multiple* mobility edges. This may be contrasted with the development of a single mobility edge due next nearest neighbor (NNN) hopping in the absence of V [10–13]. The development of mobility edges due to V can also be seen in Fig. 5, where we have plotted the single particle spectrum and the corresponding IPR values as a function of λ for $V = 0.5$. One also notices that beyond $\lambda > 2$, all the states are localized. Further, while λ shifts more levels in to the central gap region, the V has the opposite effect. Note also the development of multiple mobility edges in some range of values of λ. We also find that the delocalization to localization transition or vice versa for any energy eigen-state is associated with a rise in the single particle density of states (DOS). The variations of the DOS with V for two eigen-energies are also shown in Fig. 3. It is seen that whenever S drops or rises rapidly signaling a delocalization–localization transition or vice versa the DOS shows a peak. At this juncture we are unable to provide an analytical explanation of these results. Clearly, the superlattice potential destroys the self-duality of the AA model, and a complex interplay of the AA quasi-disorder and the superlattice potential lead to new qualitative features like the re-entrant localization–delocalization transition and the development of multiple mobility edges. Considering cold atom experiments in mind, we have checked the effect of an overall harmonic confining potential. In this case the system Hamiltonian becomes

$$\tilde{H} = H + \sum_i k i^2 c_i^\dagger c_i, \tag{6}$$