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Solitons and antisolitons on bounded surfaces

A. Ludu

Northwestern State University, Department of Chemistry and Physics, Natchitoches, LA 71497, USA

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Abstract

We generalize the one-dimensional KdV equation for an inviscid incompressible irrotational fluid layer with free surface, finite depth, and finite boundary conditions. We study the nonlinear dynamics of a fluid of arbitrary depth in a bounded domain. By introducing a special relation between the asymptotic limit of the potential of velocities at the bottom and the surface equation we obtain an infinite order PDE. The dispersion relation of the linearized equation is the well known capillarity-gravity dispersion relation for arbitrary depth. This generalized equation can be written as a differential-difference expression, and a class of traveling waves solutions in terms of power series expansion with coefficients satisfying a nonlinear recursion relation is obtained. In the limit of infinite long shallow water we recover the KdV equation together with its one-soliton solution. This generalized equation provides higher order (nonlinear) dispersion terms that do not cancel in the limit $B_0 = 1/3$. Consequently this equation can be used for investigation of soliton-antisoliton transition when the depth of the layer is closed to the critical one.

Keywords: Solitons; Antisoliton; Bounded surface

1. Introduction

Shape deformations are important for an understanding of diverse many-body systems like the dynamics of suspended liquid droplets [1], long-lying excitations of atomic nuclei or “rotation-vibration” excitations of deformed nuclei and their fission or cluster emission modes [2] hydrodynamics of vortex patches [3], evolution of atmospheric plasma clouds [4], formation of patterns in magnetic fluids and superconductors [5], electronic droplets and quantum Hall effect[6], as well as resonant formation of symmetric vortex waves [7], etc.

E-mail address: ludua@nsula.edu.

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The theoretical description of such systems is often realized in terms of collective modes, such as large amplitude collective oscillations in nuclei, sound waves in solids, collective excitations in BEC [8], plasmons in charged systems, or surface modes in flagellum-cell membrane swimming of living cells [9]. Collective modes are especially important when their energies are lower than competing singe-particle degrees of freedom. Sometimes, however, single-particle or collective modes in the bulk of a system show particularly dense or sparse density of energy levels. Systems of latter type are often referred as incompressible [1–7], and for such systems concentrating on the motion of the boundary of the system has two advantages: simpler analytical treatments of lower dimension system, and serious reduction in the numerical calculations [5–7].

The main property of the KdV equation is the equal importance of nonlinear and dispersion effects. Some of its most important solutions, like solitary waves, are single, localized, and non-dispersive structures that have a localized finite energy density. Among these solutions, the solitons are the solitary waves with special added requirements concerning their behavior at infinity ($x \to \infty, t \to \infty$) and having special properties associated with scattering with other such single-solitary wave solutions. Several extended and complex methods have been developed to study and to solve the KdV equation. Among these there is the inverse scattering theory (IST), theoretical group methods, numerical approaches. Due to its properties the KdV equation, and its quadratic extension mKdV, were the source of many applications and results in a large area of non-linear physics (for a review see [11] and the references herein).

In the present paper we use only one of the two well-known necessary conditions for obtaining the KdV equation from a one-dimensional shallow water channel, i.e.only the smallness of the amplitude of the soliton a with respect to the depth of the channel, h. This relation $a/h \ll 1$ is the only condition we use, and we take h as an arbitrary parameter. Traditionally, in order to obtain the KdV equation a second condition is imposed, i.e. the depth h of the channel should be smaller than the half-width λ of the solitary wave. In the following generalized approach we do not impose this second condition or the assumption of infinite length channel, the channel length L becomes an arbitrary parameter. We are actually investigating solutions for nonlinear waves in finite dimension domains (e.g. pools instead of channels) by replacing the fixed-length boundary condition with a periodic requirement. This approach can be used further for other bounded surfaces like liquid drops or liquid shells, bubbles, membranes, etc. This generalizations is important in applications where the height of the perturbation may be comparable to the radius (clusters formation, spherical patterns) where the model of a shallow fluid layer is inappropriate. This different starting point leads us to a new type of equation which generalizes in some sense the KdV equation (higher order in the derivatives and higher order nonlinearity), and still can reduce to the shallow liquid case.

One result occurring form this generalized form of KdV is related to the differential-difference equations theory. We can introduce a physical interpretation for the translation operator by relating it to the depth of the fluid layer. We take into account a uniform force field (like electric or gravitational field) and the surface tension acting on the free surface of the fluid. This implies that the KdV structure arises in the first order of the smallness parameter, so one doesn't have to rely on second order terms.

Another result occurring from this generalized KdV equation is related to the $B_0 \simeq \frac{1}{3}$ limit. In this case, the so called soliton-antisoliton transition limit, the traditional η_{xxx} linear dispersion term of the KdV equation vanishes and the solutions tend to blow up or become unstable. There is always a need for higher order terms which take over the dispersion