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Domain-wall crosses and propellers in binary Bose–Einstein condensates

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Abstract

For two-dimensional condensates, we introduce patterns formed by intersection of domain-walls (DWs) between immiscible species. Both symmetric and asymmetric cases are investigated, with equal or different numbers $N_{1,2}$ of atoms in the two species. The case of a rotating trap is considered too. We identify stability regions of the fundamental quiescent "DW crosses" and rotating "DW propellers", both symmetric and antisymmetric ones. In particular, the propellers are stable in a finite interval of the rotation frequencies, and asymmetric structures are stable in a finite interval of the values of N_1/N_2. The evolution of unstable patterns is also investigated. All the higher-order patterns, produced by the intersection of more than two DWs, are found to be unstable, rearranging themselves into the fundamental ones.

Keywords: Domain-wall; Soliton; Stability; Matter waves; Bose–Einstein condensation

1. Introduction

Mixtures of Bose–Einstein condensates (BECs) have attracted a lot of attention as media in which new types of stable dynamical structures are possible, the most generic ones being domain-walls (DW)

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separating immiscible BEC species [1]. Originally, the DWs were studied in models of effectively one-dimensional (1D) BECs. As quasi-1D objects, they can be extended into the 2D geometry, provided that they are stable against transverse perturbations. Such a natural extension has been studied in various theoretical works [2] and it has been shown that radially symmetric DWs, separating two concentric BEC species (the less repulsive component being in the middle of the trap and the more repulsive one forming an outer shell) can be formed. Nevertheless, in the same works, it has also been predicted that upon changing the involved parameters (such as the inter-species scattering length, strength of trap frequencies, number of atoms) a change of shape of the two species may occur, i.e., the DWs are transformed from circular to straight ones (which are stable as well). The latter possibility suggests to consider more sophisticated 2D configurations, such as a “DW cross” formed by intersection of two DWs; earlier, intersections between DWs (alias “grain boundaries”) were investigated in models of dissipative media (such as thermal convection in binary fluids), which are described by coupled Ginzburg-Landau equations [3].

The first purpose of this work is to construct DW-cross patterns for the two-component, 2D BECs with positive scattering lengths (i.e., with the repulsive interaction), loaded into the isotropic magnetic trap. We introduce two different types of the crosses, symmetric and asymmetric ones, depending on the ratio of the number of atoms in the two species.

The next step is to consider the same type of the patterns in a *rotating trap*. It is known that rotation produces a strong effect on stability of various trapped patterns [4], including the case of binary BECs [5]. The rotating DW cross seems like a propeller, which may be symmetric or asymmetric. We identify stability regions for quiescent and rotating symmetric and asymmetric crosses (which turn out to be large), and also investigate the evolution of unstable crosses. All multi-hand crosses, formed by the intersection of more than two DWs, are shown to be unstable. Thus, a new type of robust patterns in binary immiscible BECs is proposed.

One particularly appealing feature of these structures, as we will discuss, is the feasibility of their experimental realization using currently available settings. It is important to note here that there already exists a number of experimental setups currently examining binary BECs. In particular, experimental results have been reported for mixtures of different spin states of ^{87}Rb [6,7] and sodium condensates [8]. Efforts were made to create two-component BECs with different atomic species, such as ^{41}K–^{87}Rb [9] and ^{7}Li–^{133}Cs [10].

2. The model

At zero temperature, two-component rotating repulsive BECs are described by a system of two coupled Gross–Pitaevskii (GP) equations of the following form [11]:

$$i\frac{\partial\psi_j}{\partial t} = \left[\hat{H}_j + \sum_{k=1,2} g_{jk}|\psi_k|^2\right]\psi_j, \qquad j = 1 \text{ and } 2, \tag{1}$$

where ψ_j are the mean-field wave functions of the two species, normalized so that $\int|\psi_j|^2\mathrm{d}\boldsymbol{r}$ gives the number of atoms N_j, the single-atom Hamiltonians are $\hat{H}_j = -(^2/2m_j)\nabla^2 + V_j - \omega_L L_z$ (m_j is the atomic mass), V_j the trapping potential, and $L_z = i(x\partial_y - y\partial_x)$ (the subscripts stand for the partial derivatives) is the Hermitian operator that represents the z-component of the angular momentum (ω_L is the angular