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Realistic forecasting of groundwater level, based on the eigenstructure of aquifer dynamics

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Abstract

Short-term management of groundwater resources, especially during droughts, can be assisted by forecasts of groundwater levels. Such forecasts need to account for the dynamic response of the aquifer to likely recharge scenarios and recent but unknown abstractions. This paper describes a method for formulating ARMAX forecast equations from a linear system description based on the eigenvalues and eigenvectors (eigenstructure) of aquifer dynamics. For the piezometric response of a heterogeneous aquifer to a fixed spatial distribution of land surface recharge, with time-varying magnitude, only a few eigenvalues are significant. The resulting model is demonstrated with monthly values of land surface recharge and groundwater level data from a 2000 km^2 alluvial aquifer in Canterbury, New Zealand.

Keywords: Groundwater forecasting; Aquifer dynamics; Eigenvalue; ARMAX

1. Introduction

Groundwater storage in aquifers may be considered, in the natural state, as the dynamic balance between recharge, driven by climatic processes, and discharge to surface waters. Most of the dynamic behaviour is caused by variations in recharge through the land surface rather than from rivers. This balance is modified by groundwater abstraction for human use. During periods of climatic drought, land surface recharge is zero and groundwater levels decline at a rate determined by natural storage-discharge dynamics and the effect of increased abstractions for purposes such as irrigation. Management of the resource under these conditions can be assisted by short-term forecasts of groundwater levels [1]. This description, as a

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basis for the following model development, assumes that natural discharge is not affected significantly by climate. Therefore, groundwater systems for which discharge is primarily by evaporation of shallow groundwater in lowland areas would require modification of the following approach.

Forecasting of the uncertainties and dynamics of many economic and natural processes is based on the mathematics of time-series analysis (e.g., [4]). The values of observed states are considered either as univariate stochastic processes, driven by random inputs, or as bivariate processes that include other observed inputs. This essentially "black box" approach has been applied to groundwater fluctuations [6], in univariate form as well as in bivariate form with rainfall as the input variable. Ahn [1] used Kalman filter updating of a time-series model for groundwater levels at multiple sites in a multi-layered aquifer system.

Time-series analysis can also be considered as a statistical approach to identifying linearised descriptions of physical processes (e.g., [7]). Bidwell et al. [2] applied the method of [7] to a simplified conceptual model of groundwater level variations in response to recharge estimated as soil-water drainage from a water balance model.

Most mathematical descriptions of dynamic groundwater behaviour are in the form of linearised partial differential equations. This spatially-distributed linear process can also be represented as a linear system of interconnected discrete-space components, such as finite difference or finite element schemes, or as a system of conceptual linear components based on the eigenvalue solution to the distributed process [5]. Application of time-series analysis to this multi-component system description of groundwater dynamics offers the potential for completely linking aquifer properties, boundary conditions, recharge processes and external stresses, to the stochastic forecasting equations. This paper describes an approach that matches the stochastic difference equation models of time-series analysis to the physically based, linear system, groundwater model of [5].

2. Time-series models

The dynamic behaviour of many natural systems can be modelled in terms of stochastic linear difference equations, for situations where observations about the system are available at regular time intervals. The general structure of a difference equation that describes the dynamic relationship between an input time series X_n and an output series Y_n is:

$$Y_n = a_1 Y_{n-1} + a_2 Y_{n-2} + \cdots + a_p Y_{n-p} + b_1 X_n + b_2 X_{n-1} + \cdots + b_q X_{n-q+1} \tag{1}$$

Eq. (1) relates the output series to past output values (autoregressive) and a "moving average" of present and past input values. This autoregressive, moving-average (ARMA) structure requires fewer model parameters than the equivalent purely AR or MA form. By means of the z-operator (discrete-time equivalent of the Laplace operator) in its time-shift form $z^{-m} Y_n \equiv Y_{n-m}$, (1) can be expressed as the transform:

$$Y_n = \frac{b_1 + b_2 z^{-1} + \cdots + b_q z^{-q+1}}{1 - a_1 z^{-1} - a_2 z^{-2} - \cdots - a_p z^{-p}} X_n = \frac{B(z^{-1})}{A(z^{-1})} X_n \tag{2}$$

Traditional time-series analysis [4] considers a univariate stochastic series, after removal of trends and seasonal effects, as the output N_n from an ARMA model (2) with uncorrelated noise e_n as input.