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Roles of capital flow on the stability of a market system[☆]

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HIGHLIGHTS

- We model the capital flow in financial markets based on the Heston model and recycled noises.
- Good agreements between the new model and real data have been found.
- The mean escape time (MET) has been analyzed in capital inflow and outflow, respectively.
- The non-monotonic behaviors have been observed in the behaviors of MET versus delay time or rate of capital flow.

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ABSTRACT

The roles of capital flow in an ensemble composed of sub-markets are investigated. A modified Heston model and recycled noises are employed to describe the dynamics of stock price and capital flow in the ensemble, respectively. The mean escape times of two sub-markets with a cubic nonlinearity are calculated by using numerical simulation. The results evidence that (i) there is a worst delay time or rate of capital inflow concerning the minimal stability of stock price and an optimal delay time or rate of capital outflow concerning the maximal stability of stock price when $\lambda \le 0$ (λ denotes strength of correlation between two Wiener processes of the stock price and the volatility); (ii) when $\lambda > 0$, the stability of stock price is maximally enhanced by an optimal delay time or rate of capital inflow and reduced by a worst rate of capital outflow, but monotonously strengthened by delay time of capital outflow.

1. Introduction

A geometric Brownian motion that is employed to describe the stochastic dynamics of stock prices is a commonly used model in "econophysics" [1–4]. However, this model is not consistent with some statistical characteristics of the actual financial data [3,4], for instance, the fat tails [5,6], long range memory and clustering of volatility [7]. To this end, some valuable models have developed to reveal statistical properties of stock prices, for instance, an analytic approach to stock price distributions with stochastic volatility [8], the Black–Scholes option pricing model [9], arbitrage-free model [10], ARCH model [11], GARCH model [12], and Heston model [13]. Particularly, the Heston model demonstrates well the statistical

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characteristics of the actual financial data, for example, the probability distribution of returns for three stock-market indexes (e.g., Nasdaq, S&P500, and Dow–Jones) [14], the exponential distribution of financial returns obtained from the actual financial data [15], the probability density distribution of the logarithmic returns of the empirical high-frequency data of German DAX and its stocks [16], and the typical price fluctuations of the Brazilian São Paulo Stock Exchange Index [17]. Afterwards, the Heston model has been widely used to analyze dynamics of stock price in financial markets. Bonanno, Valenti and Spagnolo [18,19] analyzed the mean escape time in a modified Heston model with a cubic nonlinearity and obtained stabilizing effect of noise. Valenti et al. [20] and Spagnolo et al. [21] discussed the statistical properties of the escape times or hitting times for stock price returns in different models. Masoliver and Perelló [22,23] obtained the exact expressions for the survival probability as well as the mean exit time and a high risk of default as increasing the strength of the volatility fluctuations by solving the escape problem for the Heston random diffusion model. The effect of the delay time in Heston model has also been discussed on the stability of financial market [24] and on the risks and returns of stock investment in financial market [25].

However, the actual financial market is not an isolated system. Intuitively, from a horizontal perspective, the share of a company or financial group may be listed in multiple markets of different countries and regions, e.g., NYSE (New York Stock Exchange) and LSE (London Stock Exchange). From a vertical perspective, a company or financial group may be financing in stock markets, options markets, futures markets, physical markets, among others. The capital is exchanged in the markets. Consequently, a financial market is a complex and open subsystem, and these markets constitute an ensemble. When funds are circulated in these sub-markets, the stock price is impacted, i.e., interactions in actual financial markets are obvious. Meanwhile, the capital flow has been discussed in a two-component economic dynamical system [26], the bubble on the US markets [27], and quantum market games [28]. To describe capital flow in financial markets, from a physical point of view, we employ the recycled noise which is characterized by time delay and time correlation. Moreover, the recycled noise has commendably described control or spurious signals being transmitted across a variety of system components, and has been used in numerous fields [29–32]. For example, based on numerical simulations of a realistic recycling procedure, the feedback system has worked like a passive Maxwell's demon and induced a net current depending on both the delay and the autocorrelation times of the noise signals, when the recycled noise is multiplicatively coupled to the substrate of system [29]. Borromeo and Marchesoni [30] found the stochastic synchronization by numerical simulation in a bistable system via noise recycling. Ma, Hou and Xin [31] studied the effect of recycled noise by focusing on the performance of noise induced oscillations and coherence resonance, and Ma and Gao [32] also discussed the control effect of recycled noise in an excitable FitzHugh–Nagumo system. Hence, the recycled noise is a good approach to describe capital flow in the markets.

This paper employs a modified Heston model [18] with recycled noises [29] to analyze statistical properties of stock price returns (e.g., see Section 2.1) and the stability of stock market via numerical simulations of the mean escape time (e.g., see Section 2.2). Empirical analysis is obtained in Section 3. A brief concluding remark is given in Section 4.

2. The Heston model with recycled noise

The Heston model can be represented by the following system of coupled stochastic differential equations [13,33]:

$$\begin{aligned} \mathrm{d}x(t) &= \left(\mu - \frac{\nu(t)}{2}\right)\mathrm{d}t + \sqrt{\nu(t)}\mathrm{d}W(t), \\ \mathrm{d}\nu(t) &= a(b-\nu(t))\mathrm{d}t + c\sqrt{\nu(t)}\mathrm{d}Z(t), \end{aligned} \tag{1}$$

where $x(t)$ is the log of stock price, μ is the growth rate, $\nu(t)$ is the volatility of stock price, a is the mean reversion of $\nu(t)$, b is the long-run variance, c is the amplitude of volatility fluctuations often called the *volatility of volatility*. The deterministic solution of the $\nu(t)$ process has an exponential transient with characteristic time equal to a^{-1}, after which the process tends to its asymptotic value b [34]. In Eq. (1), $W(t)$ and $Z(t)$ are two correlated Wiener processes with the following statistical properties:

$$\begin{aligned} &\langle \mathrm{d}W(t)\rangle = \langle \mathrm{d}Z(t)\rangle = 0, \\ &\langle \mathrm{d}W(t)\mathrm{d}W(t')\rangle = \langle \mathrm{d}Z(t)\mathrm{d}Z(t')\rangle = \delta(t-t')\mathrm{d}t, \\ &\langle \mathrm{d}W(t)\mathrm{d}Z(t')\rangle = \langle \mathrm{d}Z(t')\mathrm{d}W(t')\rangle = \lambda\delta(t-t')\mathrm{d}t, \end{aligned} \tag{2}$$

where λ is the cross correlation coefficient between $W(t)$ and $Z(t)$.

Taking into account interactions in actual financial markets, they can be represented as an ensemble consisting of n open sub-markets. Additionally, capital is exchanged in the ensemble. At the same time, each sub-market is influenced by the capital flow of the ensemble, i.e., $\mathrm{d}W(t)$ and $\mathrm{d}Z(t)$ are influenced by capital flow consisting of the capital inflow, capital outflow and intrinsic capital. Hence, Eq. (1) can be rewritten as

$$\begin{aligned} \mathrm{d}x_i(t) &= \left(\mu_i - \frac{\nu_i(t)}{2}\right)\mathrm{d}t + \sqrt{\nu_i(t)}\mathrm{d}W_i(t), \\ \mathrm{d}\nu_i(t) &= a_i(b_i-\nu_i(t))\mathrm{d}t + c_i\sqrt{\nu_i(t)}\mathrm{d}Z_i(t), \end{aligned} \tag{3}$$