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New classes of Lorenz curves by maximizing Tsallis entropy under mean and Gini equality and inequality constraints

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HIGHLIGHTS

- The Tsallis entropy measure is used to construct new Lorenz curves for modeling income distribution.
- The maximum entropy principle is used, under mean and Gini index equality and inequality constraints.
- The entropic approach enables deriving maximal entropy Lorenz curves with mean and Gini index constraints.
- The inequality constraints approach is new and more realistic, since it allows a greater degree of flexibility.
- The paper extends recent results obtained in this field.

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ABSTRACT

In this paper, by using the entropy maximization principle with Tsallis entropy, new distribution families for modeling the income distribution are derived. Also, new classes of Lorenz curves are obtained by applying the entropy maximization principle with Tsallis entropy, under mean and Gini index equality and inequality constraints.

1. Introduction

The Lorenz curve, introduced by Max Lorenz in 1905 [1], represents an important instrument for analyzing data from information, computer and cognitive sciences and also for describing data from economics, especially income and wealth data [2–4]. In economics, the Lorenz curve plots the cumulative proportion of total income gained against the cumulative proportion of the population arranged according to increasing level of income or the cumulative proportion of the total productivity against the cumulative proportion of sources. The Lorenz curve has been applied in several domains of social sciences, for the study of business concentration [5] or of the distribution of firm sizes [6]. Another instrument used in the study of the inequality of income distributions, the Gini index, proposed by Corrado Gini in 1912, evaluates the ratio of the area between the Lorenz curve and the equality line. Many other quantitative techniques have been proposed for modeling

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data arising in social sciences. We can mention here the contributions of Ausloos et al. [7], who studied long birth data series and provided an interpretation based on the statistical aspects due to population sizes.

Entropy maximization represents one of the fundamental paradigms of statistical physics, which states that systems, within the boundary of their limiting constraints, will always tend towards a state of maximal disorder, i.e. maximal entropy [8]. This principle can be used to solve problems arising in many fields. The maximal entropy principle states that, when given some information about a random variable, the least biased probability distribution is obtained by maximizing entropy subject to the given constraints. Considering a complex system whose output is a real-valued random variable, the output corresponding to the maximal entropy under the constraints of a given mean and a given standard deviation is given by the Gauss distribution. Analogously, socioeconomic models can be considered, with the aim of maximizing social equality rather than physical disorder, in the context of the distributions of income and wealth in economics and social sciences.

Holm [9] obtained a family of probability density functions corresponding to the maximal Shannon entropy under mean and Gini index constraints. Ryu [3,10] tested the performance and the utility of a inequality measure depending on the Bonferroni index by using Shannon entropy maximization and defined a bottom poor sensitive Gini coefficient by replacing income observations with their reciprocal values in the Gini coefficient. Rhode [11] obtained a dependence formula between the generalized entropy and the Lorenz curve. The conclusion drawn by Rhode states that Lorenz curve can be considered as a basis for almost the inequality measures. Riabi et al. [12] obtained a family of probability density functions corresponding to the maximum second order entropy under mean and Gini index constraints and derived the corresponding Lorenz curves. Preda and Balcau [13] used the maximum entropy criterion for the maxentropic reconstruction of some Markov chains. Toma [14,15] developed divergence-based model selection criteria using dual representations of divergences and associated minimum divergence estimators and studied the robustness of dual divergence estimators for models satisfying linear constraints. Miranskyy et al. [16] investigated the application of both Shannon and extended entropies, such as Landsberg–Vedral, Renyi and Tsallis entropies to the classification of traces related to various software defects. In this paper we use the Tsallis entropy measure to construct new families of Lorenz curves for modeling the income distribution. Also new classes of Lorenz curves are constructed by applying the entropy maximization principle with Tsallis entropy, under mean and the Gini index equality and inequality constraints. The rest of the paper is organized as follows. In Section 2 the fundamental concepts that will be used in the next sections of the paper and the framework which provides the tools for the development of our new results are presented. In Sections 3 and 4 we state our main results. In Section 5 we provide some concluding remarks.

2. Preliminaries

2.1. *The Tsallis entropy*

The Tsallis entropy was introduced by Constantino Tsallis [17] in 1988 with the aim of generalizing the standard Boltzmann–Gibbs entropy and since then it has attracted considerable interest in the physics community as well as outside it. The use of Tsallis entropy enhances the analysis and solving of some important problems regarding financial data and phenomena modeling, such as the distribution of asset returns, derivative pricing or risk aversion [18–20]. Recent research in statistics increased the interest for using Tsallis entropy. Recently Trivellato [21,22] used the minimization of the divergence corresponding to the Tsallis entropy as a criterion to select a pricing measure in the valuation problems of incomplete markets and gave conditions on the existence and on the equivalence to the basic measure of the minimal k-entropy martingale measure. Preda et al. [23] used some general entropy measures in order to develop the method of measure selection for semi-Markov regime switching interest rate models in the framework of the Hunt–Devolder approach. The minimal entropy martingale for semi-Markov regime switching interest rate models is constructed using Tsallis and Kaniadakis entropies.

Let X be a real valued random variable, with the probability density function f.

Definition 1. The Tsallis entropy corresponding to the random variable X is defined as follows:

$$H_T(f) = \frac{1}{q-1}\left[1 - \int_{-\infty}^{\infty} f^q(x)\mathrm{d}x\right].$$

Note that for $q = 2$ we obtain the second order entropy [24] and for $q \to 1$ we obtain the relative Shannon entropy [25]. In the new theory the real parameter q was introduced for evaluating the degree of uncertainty.

Let X be the random variable which models the level of income. We suppose that X takes values in the interval $[x_0, x_1]$ and denote by f the probability density function and by F the cumulative distribution function of the random variable X, where $F(x)$ stands for the proportion of the population with income level less than or equal to x.

Now, let $\mathcal{L}$ be the class of all non-negative random variables with positive finite expectations. For a random variable X in $\mathcal{L}$ with cumulative distribution function F, we define its quantile function by

$$F^{-1}(y) = \inf\{x : F(x) \geq y\}.$$

We denote by μ the expectation of the random variable X. We introduce the definition of the Lorenz curve given by Gastwirth [26]. Let $X \in \mathcal{L}$ with cumulative distribution function F and quantile function F^{-1}. Let $p \in [0, 1]$.