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Spatiotemporal recurrences of sandpile avalanches

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HIGHLIGHTS

- Spatiotemporal recurrences in the sandpile model were investigated.
- We connect avalanches with nearest or farthest later events.
- We observe previously unreported scale-free regimes in space and time.
- Connecting farthest events also resulted in scale-free in-degree distributions.

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ABSTRACT

We study the space and time properties of avalanches in a continuous sandpile model by constructing a temporally directed network linking together the recurrent avalanche events based on their spatial separation. We use two different criteria for network construction: a later event is connected to a previous one if it is either nearest or farthest from it among all the later events. With this, we observe scale-free regimes emerge as characterized by the following power-law exponents: (a) $\alpha = 1.7$ for the avalanche size distributions; (b) $\beta_F = 2.1$ in the in-degree distribution of farthest recurrences; (c) $\delta = 1$ for the separation distances; and (d) $\gamma = 1$ for the temporal separations of recurrences. Our results agree with earlier observations that describe the sandpile avalanches as repulsive events, i.e. the next avalanche is more likely to be physically separated from an earlier one. These observations, which are not captured by usual interoccurrence statistics and by random connection mechanisms, suggest an underlying spatiotemporal organization in the sandpile that makes it useful for modeling real-world systems.

1. Introduction

The sandpile model and its variants, originally intended as representative systems for illustrating self-organized criticality (SOC) [1], have now been utilized for modeling systems in the natural setting believed to be exhibiting SOC characteristics. These sandpile-based approaches use the avalanches in the grid as representative of the event size of the natural system in consideration. In many cases, therefore, the efficacy of a model is primarily gauged by its ability to replicate the power-law and “rollover” regimes in the event size distributions, like the Gutenberg–Richter law of earthquake energies [2–4] and the volume and area distributions of landslides [5–7]. It is important to note, however, that such natural hazards have an inherent spatial and temporal dimension; earthquakes, in particular, show strong spatiotemporal clustering [8], while landslides tend to occur at susceptible regions, as characterized by the slope properties [9]. Another point of interest is in the correlation among these events, which are believed to exhibit strong memory. It is therefore of particular interest to see whether these spatiotemporal properties and event correlations are also captured by sandpile-based approaches.

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To this end, one may use the statistical distributions of separation distances and recurrence times between successive avalanches in the sandpile grid. For a sandpile triggered at random locations at discrete times, these distributions are expected to be of random nature due to the independence of the triggering mechanism. Imposing a threshold magnitude for taking interevent properties have resulted in fat-tailed distributions that closely resemble the actual statistics obtained in experiments and field data [6,10]. These analyses, however, neglect the faint yet more frequent avalanche events, whose collective effect can give rise to the larger avalanches in the system.

In this work, we attempt to uncover the underlying spatiotemporal correlations of sandpile avalanches using complex network techniques. Our approach is based on a generalized method for constructing spatiotemporal recurrence networks, wherein two events can have a temporally directed link if a later event breaks a spatial distance record relative to a previous one [11]. First applied to seismicity, the method revealed an underlying causal structure when the next event is nearest to a preceding one [12]. Here, apart from this criterion, we are guided by the fact that extreme avalanches tend to be repulsive [13], which merits the use of the farthest-distance criterion for connecting avalanche events. Our results reveal scale-invariant spatiotemporal features of sandpile avalanches, which are not readily observable using successive-event characterizations.

2. Model parameters

To be of practical use for capturing the statistical features of hazard events in nature, the original sandpile has to incorporate modifications while still maintaining the SOC characteristics of the grid. Here, we implement the most straightforward modification to the sandpile rules by preserving the discrete nature of time and space while using continuous "energy" values for each cell, similar to that of Zhang [14]. We used a sandpile grid of dimensions $L \times L$, where $L = 256$, initialized to have states $\sigma(x, y, 0) \in [0, \sigma_{max})$, where σ_{max} is set to unity. Every discrete time step, a random site (i, j) in the pile receives an increase in energy by an amount ν,

$$\sigma(i, j, t+1) \rightarrow \sigma(i, j, t) + \nu \tag{1}$$

where $\nu = \sigma_{max}/1000$, small enough to produce SOC in the grid, as determined by large-scale implementations by Lübeck [15]. To compensate for this relatively slow rate of external triggering, we conducted numerical runs with long iteration periods $t_{max} = 10^7$ while neglecting the first 10^5 iterations to remove transient effects.

Triggering will eventually drive certain cells to instability when their state $\sigma(i, j) \geq \sigma_{max}$. Under such conditions, time is frozen and no new external trigger ν is introduced while the unstable cell is allowed to relax and redistribute its state into its four nearest neighbors $nn = \{(i \pm 1, j), (i, j \pm 1)\}$ according to the following rules:

$$\sigma(nn) \rightarrow \sigma(nn) + \frac{\sigma(i, j)}{4} \tag{2}$$

$$\sigma(i, j) \rightarrow 0 \tag{3}$$

which, in turn, can cause a cascade of relaxation events when the nearest-neighbors themselves are driven to instability. This cascade mechanism is responsible for the emergence of scale-free distribution of avalanche sizes.

In this work, we track the time series of avalanche sizes $A(t)$ which is the number of sites affected by a single triggering event and possible resulting cascades. In addition, we also recover the location of the original instability (i, j) to complete the spatiotemporal characterization.

3. Complex network of recurrent avalanches

To describe the spatiotemporal relationships among avalanche events without an a priori knowledge of the underlying structure and without imposing arbitrarily-defined windows, we use the method first proposed by Davidsen et al. [11] that constructs directed links between events separated in time based on their relative separation distances. While it can be argued that many systems in nature exhibit strong memory, thereby requiring all later events to receive a connection from all previous ones, one can use the separation distance between events as an additional condition for establishing possible relationship between events. In Davidsen et al. [12], the case of seismicity is investigated, and the strong clustering behavior merits that a later event will be connected to a previous event if it breaks the record of being the *nearest* to it. In view of previous results that show that large sandpile avalanches tend to be spatially repulsive [13], we attempt to extend the analysis by also considering the complex network constructed when the later event is connected to a previous one only if it is *farthest* to it. Fig. 1 shows the schematic representation of the two network-construction schemes for the first five events (after the transient time) in a sample sandpile run.

The temporal condition of connecting a previous event to a succeeding event (and not backwards) allows for $k_{max} = n(n-1)/2$ links for n events. Imposing the spatial criterion of nearest or farthest separations necessarily decrease the number of links for a two-dimensional space of the sandpile with random triggering rules. In Fig. 1, the broken lines denote possible temporal connections, while the arrows denote those that met the spatial criterion of being the nearest (Fig. 1(a)) or farthest (Fig. 1(b)) record. Additionally, the colors of the lines denote the origin of the connections, i.e. which previous avalanche origin has had a record broken. Apart from the obvious similarity in the successive-event connections, the two