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Asymmetric optimal-velocity car-following model

Xihua Xu [a,b,*], John Pang [c], Christopher Monterola [c]

[a] Department of Mathematics, National University of Singapore, 119076, Singapore
[b] Beijing Computational Science Research Center, Beijing 100084, PR China
[c] Complex Systems Group, Institute of High Performance Computing, A* STAR, 138632, Singapore

HIGHLIGHTS

- A new asymmetric optimal-velocity car-following model is proposed.
- The asymmetry is represented by the exponential function with an asymmetrical factor.
- The deceleration is stronger than acceleration with the same velocity difference.
- Unrealistically acceleration disappears when the velocity difference becomes large.
- The strength of interaction between clusters is increasing with the asymmetry factor.

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ABSTRACT

Taking the asymmetric characteristic of the velocity differences of vehicles into account, we present an asymmetric optimal velocity model for a car-following theory. The asymmetry between the acceleration and the deceleration is represented by the exponential function with an asymmetrical factor, which agrees with the published experiment. This model avoids the disadvantage of the unrealistically high acceleration appearing in previous models when the velocity difference becomes large. This model is simple and only has two independent parameters. The linear stability condition is derived and the phase transition of the traffic flow appears beyond the critical density. The strength of interaction between clusters is shown to increase with the asymmetry factor in our model.

1. Introduction

During last several decades, traffic problems have been modeled mathematically and physically in various contexts. Many investigations have been done with different points of view to consider the various aspects of traffic phenomena.

One interesting phenomena of traffic flow is the propagation of wide moving jams [1]. In all empirical observations of real traffic flow on highways wide moving jams emerge in synchronized flow ($S \to J$ transition), rather than in free traffic flow [2,3]. This result of empirical observations of real traffic has been explained in three-phase theory by instability of synchronized flow [2,3]. This instability of synchronized flow can also explain the formation of wide moving jams in traffic experiments conducted in a road circuit [4,5]. Indeed, as shown in Ref. [6], the homogeneous traffic flow in the experiments [4,5] is related to synchronized flow, rather than to free traffic flow. This congestion phenomena is regarded as the instability and the phase transition of a dynamical system, and has been modeled using various approaches, see for example Refs. [7–10].

* Corresponding author at: Beijing Computational Science Research Center, Beijing 100084, PR China. Tel.: +65 9036 0067.
E-mail address: matxx@nus.edu.sg (X. Xu).

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The optimal velocity (OV) model by Bando et al. [11] cannot describe synchronized flow. Nevertheless, in this paper to simulate some features of moving jam emergence in traffic experiments conducted in a road circuit [4,5], with can use the OV model. This methodology of a numerical study of moving jam emergence can be explained by the following fact mentioned in a recent review [12]: Although a diverse variety of complex spatiotemporal phenomena in synchronized flow have been found in the three phase theory [2,3], one of the most important of them – moving jam emergence in synchronized flow ($S \rightarrow J$ transition) – is explained by the over-deceleration effect (driver's over-reaction) due to a driver's reaction time, i.e., by the same traffic flow instability discovered in the GM model. This explains many similar results of the three-phase theory and the GM model class in the description of the features of moving jam propagation in congested traffic.

For the OV model, the velocity v_n of the nth car takes the following form:

$$\frac{\mathrm{d}v_n}{\mathrm{d}t} = \kappa[V(h_n) - v_n], \tag{1}$$

where $h_n = x_{n+1} - x_n$ is the headway, x_n is the position of nth car, κ is the coefficient of sensitivity and $V(h)$ is the distance-dependent optimal velocity that the vehicles adapt to. As a result of the distinctive feature in representing real traffic flow characteristics such as the evolution of the traffic congestion, there exists a strong push to study and develop this model in a realistic way.

Realizing that the OV model may result in impractical high acceleration and unrealistic deceleration from the experimental study, Helbing and Tilch [13] added a new term representing the impact of the negative difference in velocity to the OV model and developed a generalized force (GF) model as:

$$\frac{\mathrm{d}v_n}{\mathrm{d}t} = \kappa[V(h_n) - v_n] + \lambda\Theta(-\Delta v_n)\Delta v_n, \tag{2}$$

where Θ is the Heaviside function and Δv_n is the relative velocity, i.e., $\Delta v_n = v_{n+1} - v_n$. A new optimal velocity function was also proposed with respect to the empirical data as:

$$V(h) = V_1 + V_2 \tanh[C_1(h - l_c) - C_2], \tag{3}$$

where l_c is the length of the vehicles taken as 5 m in simulations. The resulting optimal parameter values are $V_1 = 6.75$ m/s, $V_2 = 7.91$ m/s, $C_1 = 0.13\ \mathrm{m}^{-1}$, and $C_2 = 1.57$. Generally, this optimal velocity function is monotonically increasing with headway h, and bounded from above and below by the maximum and minimum velocity of the car.

Noting that the GF model only considers the case where the velocity of the following vehicle is larger than that of the leading vehicle, Jiang et al. [14] proposed a full velocity difference (FVD) model that takes both positive and negative velocity difference into account:

$$\frac{\mathrm{d}v_n}{\mathrm{d}t} = \kappa[V(h_n) - v_n] + \lambda\Delta v_n. \tag{4}$$

A major deficiency of this model lies in the fact that it models the velocity differences of vehicles symmetrically, which is unrealistic. Gong et al. [15] modified this expression to consider the effects of asymmetric acceleration and deceleration in their asymmetrical full velocity difference (AFVD) model:

$$\frac{\mathrm{d}v_n}{\mathrm{d}t} = \kappa[V(h_n) - v_n] + \lambda_1\Theta(-\Delta v_n)\Delta v_n + \lambda_2\Theta(\Delta v_n)\Delta v_n. \tag{5}$$

To represent different acceleration and deceleration, Eq. (5) applies two sensitivity coefficients to model the velocity difference. It is assumed that the vehicles' capability in deceleration is greater than in acceleration, corresponding to $\lambda_1 > \lambda_2$ in their model.

Taking this asymmetric characteristic into account, Shamoto et al. [16] proposed a car-following model based on the experiments:

$$\frac{\mathrm{d}v_n}{\mathrm{d}t} = a - b\frac{v_n}{(h_n - d)^2}\exp(-c\Delta v_n) - \gamma v_n, \tag{6}$$

here a, b, c, d, γ are positive parameters. For this model, multiple 'stop-and-go' (SAG) waves are hard to merge, because h_n is approximately equal to d in the jam flow and $v_n/(h_n - d)^2$ becomes infinite. The acceleration of the car cannot be maintained constantly resulting in few cars in the cluster, and the wide moving jam cannot be formed.

2. Asymmetric optimal-velocity car-following model

As the relative velocity Δv_n increases, the acceleration of a car should have a realistic physical limit, even when the relative velocity becomes infinite ($\Delta v_n \rightarrow \infty$). In addition, drivers will apply emergency braking to avoid the collision when relative velocity is negative and becomes smaller. Therefore, the relationship between relative velocity and acceleration (deceleration) is in general nonlinear in reality as demonstrated by actual experiments [16]. FVD models and AFVD models are formulated by linearly adding the relative velocity term, in disagreement with the experimental data.

Building on these observations, we have determined an asymmetric optimal velocity (AOV) car-following model as follows:

$$\frac{\mathrm{d}v_n}{\mathrm{d}t} = \kappa\left[V(h_n) - v_n + \Delta v_n \exp(-\mu\Delta v_n)\right], \tag{7}$$