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Effect of degree correlation on exact controllability of multiplex networks

Sen Nie [a], Xuwen Wang [a,*], Binghong Wang [a,b,c,*]

[a] Department of Modern Physics, University of Science and Technology of China, Hefei, Anhui 230026, PR China
[b] College of Physics and Electronic Information Engineering, Wenzhou University, Wenzhou, Zhejiang 325035, PR China
[c] School of Science, Southwest University of Science and Technology, Mianyang, Sichuan 621010, PR China

HIGHLIGHTS

- Interconnections' correlation influences controllability of multiplex networks.
- Assortative multiplex networks are easier to control for sparse interconnections.
- Disassortative multiplex networks are easier to control for dense interconnections.
- Controllability depicts transition with density of interconnections.

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ABSTRACT

It has been proved that the degree correlation can affect the structural controllability of directed networks. Here, we explore the effect of interconnections' correlation on the exact controllability of multiplex networks. We find that the minimal number of driver nodes decreases with correlation for lower density of interconnections. However, the controllability of networks with higher density of interconnections shows the contrary tendency. For different interconnections' correlations, controllability of multiplex networks depicts transition with the density of interconnections. For lower interconnections density, the networks with disassortative coupling patterns are harder to control. Whereas, for higher interconnections density, the networks with assortative coupling patterns are harder to control.

1. Introduction

Controllability of complex networks has been a hot topic recently with the further study of dynamical complex networks [1–4]. It focuses on the ultimate goal of studying complex networks, which is how to control the system from any initial state to any final state in finite time by external inputs [5]. Liu et al. [1] combined the complex networks with the control theory of structural controllability to obtain the minimal number of inputs which could fully control the whole directed network. More, exact controllability of networks, which is suitable for arbitrary network structure and link weights was proposed in Ref. [4]. The recent works have paid more attention on the observability [3], target control [6], control energy [7] and other significant problems [8–17]. All of these results are discussed in the isolated networks, however, it has been demonstrated that many real-world networks are coupled together to interact with each other [18,19]. For example, relationship network, emails network and other online social networks couple into the multiplex communication networks.

* Corresponding authors.
E-mail addresses: xww86@mail.ustc.edu.cn (X. Wang), bhwang@ustc.edu.cn (B. Wang).

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Ref. [20] focused on the exact controllability of multiplex networks and pointed out that the controllability of multiplex networks can be affected by the density of interconnections between layers. The multiplex networks have also been studied widely in cascading failures [21,22], evolutionary game dynamics [23,24], traffic dynamics [25] and other fields of dynamical complex networks [26–28].

Degree correlation is ubiquitous in complex networks. It describes the tendencies of nodes to connect with other nodes that have similar in- and out-degrees as themselves, respectively [29]. If the high degree nodes tend to be connected with other high degree nodes, then the networks are assortative. However, the networks in which the high degree nodes tend to be connected with low degree nodes are disassortative. To real networks, the biological and technological networks are disassortative while the social networks are usually assortative [29]. It has been demonstrated that the degree correlation affects the structural controllability of complex networks [30]. In multiplex networks, the nodes in one network couple with the nodes in the other network, but the coupling configuration is usually not random. Such as in a multiplex network coupled by power grid and computer network, an important node in power grid may prefer to connect with the important node in computer network. The different coupling patterns not only influence the robustness of multiplex networks [22], but also affect the controllability of multiplex networks.

In this paper, we focus on the degree correlation of interconnections on the exact controllability of multiplex networks. The results show that the minimal number of driver nodes decreases with correlation for sparse interconnections. But, it increases with correlation for dense interconnections. By incorporating with degree correlation, effect of interconnection's density on the exact controllability of multiplex networks is interesting. For lower density of interconnections, the network with assortative coupling pattern is easier to control, but it is hard to control for higher density of interconnections. This paper is organized as follows: Section 2 is the model, Section 3 is numerical results, and the discussion is presented in Section 4.

2. Model

2.1. Exact controllability

We consider the system coupled by two N-nodes networks with linear time-invariant dynamics. A fraction of nodes in layer-1 connect with the nodes in layer-2. For simplicity, the nodes in one layer can only interact with one node in the other layer. The dynamical equation of 2-layer networks is given by:

$$\dot{x}(t) = A^C x(t) + Bu(t) = \begin{bmatrix} A_1 & A_{12} \\ A_{21} & A_2 \end{bmatrix} x(t) + Bu(t), \tag{1}$$

where $x(t) = (x_1(t), x_2(t), \ldots, x_{2N}(t))^T$ is the state of $2N$ system at time t. $A^C_{2N\times 2N}$ is adjacent matrix, in which A_1 and A_2 represent the interaction strength between nodes of each layer, respectively. The matrices A_{12} and A_{21} describe the interconnections of two layers. $B_{2N\times M}$ is input matrix which defines how the external inputs are connected to the nodes of networks and $u(t) = (u_1(t), u_2(t), \ldots, u_M(t))^T$ is the input vector. According to Ref. [4], the minimal number of driver nodes can be exactly calculated by following equations based on the PBH rank condition [31]:

For arbitrary networks and link weights, the number of driver nodes is given by:

$$N_D = \max_i \{\mu(\lambda_i)\}, \tag{2}$$

where $\mu(\lambda_i)$ are the geometric multiplicity of λ_i of A^C, and $\lambda_i (i = 1, \ldots, l)$ are the distinct eigenvalues of A^C.

For undirected networks with arbitrary link weights, N_D is determined by:

$$N_D = \max_i \{\delta(\lambda_i)\}, \tag{3}$$

where $\delta(\lambda_i)$ are the algebraic multiplicity of A^C. Especially, Eq. (2) can be simplified as $N_D = \max\{1, 2N - rank(A^C)\}$ for large sparse networks with a small fraction of self-loops; $N_D = \max\{1, 2N - rank(wI_{2N} + A^C)\}$ for dense networks with identical link weights w.

2.2. Degree correlation

Degree correlation measures the tendencies of connections between nodes, and it can be quantified by the Pearson coefficient [32]:

$$r = \frac{M^{-1}\sum_i j_i k_i - \left[M^{-1}\sum_i 1/2(j_i + k_i)\right]^2}{M^{-1}\sum_i 1/2(j_i^2 + k_i^2) - \left[M^{-1}\sum_i 1/2(j_i + k_i)\right]^2}, \tag{4}$$

where $\sum_i \cdot$ sums over all edges, j_i and k_i are the degrees of nodes which belong to the ith edge, and M is the total number of edges. Positive value of r indicates the assortative network while the negative value of r characterizes the disassortative network.