Physica A 436 (2015) 499–508

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Context dependent preferential attachment model for complex networks

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HIGHLIGHTS

- We propose a random growing complex network model by adopting the concept of context dependent preferential attachment.
- The networks generated by this model inherit properties of real world networks, for example, power law degree distribution, small expected diameter.
- We have numerically calculated different properties of complex networks generated by the proposed model.

ARTICLE INFO

Article history:
Received 21 January 2015
Received in revised form 18 April 2015
Available online 14 May 2015

Keywords:
Context dependent preferential attachment
Degree distribution
Diameter
Cluster coefficient
Spectral radius
Algebraic connectivity

ABSTRACT

In this paper, we propose a growing random complex network model, which we call context dependent preferential attachment model (CDPAM), when the preference of a new node to get attached to old nodes is determined by the local and global property of the old nodes. We consider that local and global properties of a node as the degree and relative average degree of the node respectively. We prove that the degree distribution of complex networks generated by CDPAM follow power law with exponent lies in the interval [2,3] and the expected diameter grows logarithmically with the size of new nodes added to the initial small network. Numerical results show that the expected diameter stabilizes when alike weights to the local and global properties are assigned by the new nodes. Computing various measures including clustering coefficient, assortativity, number of triangles, algebraic connectivity, spectral radius, we show that the proposed model replicates properties of real networks when alike weights are given to local and global property. Finally, we observe that the BA model is a limiting case of CDPAM when new nodes tend to give large weight to the local property compared to the weight given to the global property during link formation.

1. Introduction

Modeling complex networks has been an active area of research in the literature due to its applications in various fields of science and technology [1–4]. Several attempts have been made to generate deterministic and random complex network models which can capture the spirit of several large scale real world networks such as social networks [5], biological networks [6], and technological networks [7]. Two prime characteristics of a large class of real networks that have been observed and established by leading scholars in the area of complex networks are power-law degree distribution of the

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http://dx.doi.org/10.1016/j.physa.2015.04.038
0378-4371/

nodes and small-world behavior of the networks [8–12]. The Erdös–Rényi (ER) model [13] is one of the first initiatives to generate random networks where the links are made by following a random procedure when a fixed number of nodes is chosen at the initial stage of the network formation. However, later it has been observed that ER model fails to represent the essence of real networks, for example, degree distribution is not a power-law. Consequently, a lot of interest has been generated to produce networks having power-law degree distributions.

One of the insightful growing random complex network models is proposed by Barabási et al. in 1999, also called BA-model [11]. In this model, a small network is chosen in the beginning of the method, then new nodes appear and get linked with the existing nodes in a probabilistic fashion which is decided by the property (degree) of the existing nodes [11,14]. The philosophy adopted here is that at each iteration, the new nodes prefer to get attached with an old node which has high degree (among all existing ones) which sometimes represents the importance of a node in social context. Interestingly, a network generated by this model has power-law degree distribution and thus the concept of scale-free networks emerged. In their seminal paper [11], Barabási et al. have also predicted that the growth and preferential attachment are jointly responsible for the emergence of the scale-free property in real networks. It has also been shown that the diameter grows approximately logarithmically with the size of the network.

Does a new node always wish to form links with important (high degree) nodes or the choice get influenced by other factors also? Moreover, if the choice gets influenced by other properties of the existing nodes, will the network be having power-law degree distribution? An evidence of a phenomena that people's choice does not depend on only one property is given in Ref. [15] supported by an empirical data (see Refs. [16–18] also). The data shows that at the time of purchasing a product, a buyer considers the background (history) of the product and relative attractiveness of the product with respect to other products in the same reference. Thus, the concept of context preferential attachment was introduced in Ref. [15].

In this paper, we propose a growing random complex network model where the probability of link formation is determined by weighted local and global property of the existing nodes. We consider that local and global properties of a node are given by the degree and relative average degree of the node in a network. Thus, we call the proposed model, the context dependent preferential attachment model (CDPAM) for complex networks. We prove that the degree distribution of complex networks generated by CDPAM follows power law $P(k) = L(k)k^{-\gamma}$ where $2 \leq \gamma \leq 3$ and $L(k) \to \alpha$ (a constant which depends on the weights given on local and global property of the nodes) as $k \to \infty$. We also prove that the expected diameter grows logarithmically with the size of the new nodes added in the network, however the growth of the expected diameter is slower than that of the BA model. The numerical simulations show that the expected diameter stabilizes when alike weights are given to the local and global property which determine the preference of link formation. In contrast to the conventional wisdom that diameter shows as a function of $\ln(\ln N)$ or $\ln N$ in real networks, the authors in Ref. [19] observed that the diameter stabilizes or shrinks as a network grows. The proposed model reveals how shrinking and increasing of diameter are related to the weights on local and global property of the nodes during expansion of the network.

A variety of mathematical and statistical measures have been proposed in the literature in order to characterize global and local structure of complex networks. We derive clustering coefficient, assortativity, number of triangles, algebraic connectivity and spectral radius for different complex networks generated by CDPAM and compare them with the same obtained from the complex networks generated by BA model. We show that CDPAM replicates properties of real networks for all these measures when alike weights are given to local and global property. Finally, we observe that the BA model is a limiting case of CDPAM when new nodes tend to give large weight to the local property compared to the weight given on the global property during link formation.

2. Context dependent preferential attachment model (CDPAM)

In this section, we propose a random complex network model which relies on the fact that the network is open i.e. a network continuously grows in time with the addition of new nodes into a fixed small network chosen in the beginning of the process [20]. It is important to notice that the link formation in BA model is biased as the link formation depends only on the high degree (importance) of the existing nodes. However, in real life individuals not only prefer to form relationship (link) with important (global property) people in society but also give importance to background (local property) of the people before making the relation. Inspired by this thought, we introduce the model as follows.

1. Growth: Starting with a small network having m_0 nodes, at every timestep add a new node with $m \leq m_0$ edges such that degree of any node in the initial network should lie between m and $2m$.
2. Context preferential attachment: Assume that $N(t)$ denotes the node set of the network after t-timestep. When a new node j appears at time $t+1$, it will get connected to a node $i \in N(t)$ with probability $p_j^i(t+1)$ given by

$$p_j^i(t+1) = \frac{\beta f_B(i) + \theta\, g(i, N(t))}{\sum\limits_{i \in N(t)} (\beta f_B(i) + \theta\, g(i, N(t)))} \tag{1}$$

where $f_B(i)$ quantifies the background (local context) of node i, $g(i, N(t))$ determines the relative advantage (global context) of a node over others in the network $N(t)$, and $\beta, \theta(<\beta)$ are the positive control parameters for the property of the nodes in $N(t)$.