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Simultaneous selection and estimation of the best treatment with respect to a control in a general linear model

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Abstract

The problem of selecting the largest treatment parameter, provided it is better than a control, and simultaneously estimating the selected treatment parameter in a general linear model is considered in the decision theoretic Bayes approach. Both cases, where the error variance is known or unknown, are included. Bayes decision rules are derived for noninformative and for normal priors. Bayes rules for noninformative priors are derived under a general loss function for designs that satisfy the BTIB condition of Bechhofer and Tamhane (Technometrics 23 (1981) 45). For unbalanced designs, a linear loss function is adopted and it is demonstrated, via simulations, that the simultaneous estimation of the selected treatment effect plays an important role in correcting an undesirable effect for the selection problem.

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Keywords: Selection and estimation; Comparisons with respect to a control; Loss function; Bayes rule; Balanced and unbalanced design.

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1. Introduction

Bechhofer et al. (1995) have treated the problem of selecting the best treatment, provided it is better than a control, in the indifference-zone approach. These and other authors consider k-independent treatment populations and an independent control population. In the present approach, a general linear model is considered instead. For such a model, we utilize the decision theoretic Bayes approach to simultaneously estimating the selected treatment effect while selecting the best treatment, provided it is better than the control. A similar problem has been considered by Bansal and Miescke (2002) for situations where there is no control.

We consider the following general linear model:

$$Y = X_1\tau + X_2\beta + \varepsilon, \tag{1}$$

where $Y(n \times 1)$ is a vector of responses, $X_1(n \times (k+1))$ and $X_2(n \times m)$ are design matrices, $\tau = (\tau_0, \tau_1, \ldots, \tau_k)^{\mathrm{T}}$ is a parameter vector consisting of the control effectτ_0 and the k treatment effects $\tau_1, \ldots, \tau_k$, and $\beta(m \times 1)$ is a vector of nuisance parameters such as block effects. We assume without loss of generality that the rank of X_2 is equal to m, and we assume that the error satisfies $\varepsilon \sim N_n(01, \sigma^2 I)$, where $1 = (1, 1, \ldots, 1)^{\mathrm{T}}$ and $\sigma^2 > 0$ is either known or unknown.

Let the parameter space be denoted by $\Theta_1 = \{\theta = (\tau^{\mathrm{T}}, \beta^{\mathrm{T}})^{\mathrm{T}} : \tau \in R^{k+1}, \beta \in R^m\}$ in the case where σ^2 is known, and by $\Theta_2 = \{(\theta, \sigma^2) : \theta \in \Theta_1, \sigma^2 > 0\}$ in the case where σ^2 is unknown. For many important special designs of model (1), not all components of τ are identifiable. We assume, however, that $\Delta_i = \tau_i - \tau_0$, $i = 1, \ldots, k$, are identifiable.

To select the treatment associated with largest treatment effect $\tau_{[k]} = \max\{\tau_1, \ldots, \tau_k\}$, provided it is better than the control, and to simultaneously estimate the value of$\Delta_s = \tau_s - \tau_0$, where τ_s is associated with the selected treatment s, say, decision rules of the form $d(y) = (s(y), e_{s(y)}(y))$, $y \in R^n$, will be considered. Here, $s : R^n \to \{0, 1, \ldots, k\}$ is the selection sub-decision rule of d, and $e_s : R^n \to R$ is the estimation sub-decision rule of d. Here e_s estimates Δ_s, where s represents the selected treatment. A selection of $s = 0$ means that the control is decided to be better than all of the k treatments, and in that case no estimation is required. Thus, for convenience we set $e_0 = 0$. It should be pointed out that based on the observation Y, $e_{s(Y)}(Y)$ has a random index $s(Y)$.

For simultaneous selection and estimation, the loss has to include two components, one for the selection and one for the estimation, cf. Gupta and Miescke (1990). It is thus natural to assume that the loss function is of the form

$$L(\tau, d) = A(\tau, s) + B(\Delta_s, e_s), \tag{2}$$

where $A(\tau, s)$ represents the loss due to selection, and $B(\Delta_s, e_s)$ represents the loss due to estimation with $B(\cdot, 0) = 0$. In the literature, most of the work in ranking and selection deals only with the loss due to selection. We will show that in the present setting, the loss due to estimation also plays an important role for the selection part of the problem in unbalanced designs. It provides, in some way, an adjustment when Δ_s, corresponding to the selected treatment, is not efficiently estimated. Special care must be taken, however, when choosing $B(\Delta_s, e_s)$. In particular, $B(\Delta_s, e_s)$ should not be too large in comparison to $A(\tau, s)$. Otherwise, as we will see later, the selection rule may force the control treatment to be selected for any observation.