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Rényi test statistics for partially observed diffusion processes

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Abstract

Diffusion processes $X(t)$ verifying the stochastic differential equation $\mathrm{d}X(t) = a\,\mathrm{d}t + b\,\mathrm{d}W(t)$, $X(t_0)=X_0$, $b > 0$, are considered for standard Wiener processes $W(t)$ in $0 < t_0 < t < T$. Problems of testing hypotheses about the parameters are analyzed when the stochastic process is partially observed. A family of test statistics is introduced on the basis of the Rényi divergence measure and their asymptotic distributions are obtained. To finish, a simulation study is given in order to compare powers of some of the introduced statistics.

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1. Introduction

Parametric statistical inference on continuous stochastic processes $X(t)$ is often a difficult task. Moreover, in many cases it is not possible to record complete information about realizations. Because of this reason it is worthwhile to have statistical procedures for partially observed stochastic processes.

Let $X(t)$ be a diffusion process verifying the stochastic differential equation $\mathrm{d}X(t) = a\,\mathrm{d}t + b\,\mathrm{d}W(t)$, $X(t_0) = X_0$, $b > 0$, where $W(t)$ is a standard Wiener process. Assume that there exist times $0 < t_0 < t_1 < \cdots < t_n = T$, where $X(t)$ is observed and define the

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increments $\Delta t_i = t_i - t_{i-1}, i = 1, 2, \dots, n$, not necessarily equal. Under these assumptions, it holds that

$$\Delta X(t_i) \triangleq X(t_i) - X(t_{i-1}) = a\Delta t_i + \eta_i, \tag{1.1}$$

where $\eta_i = b\Delta W(t_i) \sim \mathrm{N}(0, b^2\Delta t_i)$, $i = 1, 2, \dots, n$, are independent. Here and in the sequel $\sim$ is used to denote "is distributed as". Dividing by $(\Delta t_i)^{1/2}$ in both sides of equality (1.1), one gets

$$Y(t_i) \triangleq \frac{\Delta X(t_i)}{(\Delta t_i)^{1/2}} = a(\Delta t_i)^{1/2} + \frac{\eta_i}{(\Delta t_i)^{1/2}} \triangleq ah(t_i) + \varepsilon_i, \quad i = 1, 2, \dots, n,$$

where $\varepsilon_i \sim \mathrm{N}(0, b^2)$ are independent. The result of this algebra is an homoskedastic linear model $Y = ah + \varepsilon$, where $Y = (Y(t_1), \dots, Y(t_n))^{\mathrm{t}}$, $h = (h(t_1), \dots, h(t_n))^{\mathrm{t}}$ and $\varepsilon = (\varepsilon_1, \dots, \varepsilon_n)^{\mathrm{t}}$. Under these assumptions the joint probability density function of Y at $Y(t_1) = y_1, \dots, Y(t_n) = y_n$ is

$$f_{a,b^2}(y_1, \dots, y_n) = (2\pi b^2)^{-n/2} \exp\left\{-\frac{1}{2b^2}\sum_{i=1}^{n}(y_i - ah(t_i))^2\right\} \tag{1.2}$$

and the maximum likelihood estimators (MLE) of parameters are

$$\hat{a}_n = \frac{X(t_n) - X(t_0)}{t_n - t_0} \quad \text{and} \quad \hat{b}_n^2 = \frac{1}{n}\sum_{i=1}^{n}(Y(t_i) - \hat{a}_n h(t_i))^2. \tag{1.3}$$

It can be easily checked that

$$\hat{a}_n \sim \mathrm{N}\left(a, \frac{b^2}{t_n - t_0}\right) \quad \text{and} \quad \frac{n\hat{b}_n^2}{b^2} \sim \chi^2_{n-1}, \tag{1.4}$$

are stochastically independent. Furthermore, observe that $E[\hat{b}_n^2] = [(n-1)/n]b^2$, so that $\hat{b}_n^2$ is an asymptotically unbiased estimator of b^2. However, $S_{\mathrm{R}}^2 = [n/(n-1)]\hat{b}_n^2$ is unbiased for b^2 and

$$\frac{(n-1)S_{\mathrm{R}}^2}{b^2} \sim \chi^2_{n-1}.$$

By substituting (1.3) in (1.2), the estimated density

$$f_{\hat{a}_n,\hat{b}_n^2}(y_1, \dots, y_n) = (2\pi\hat{b}_n^2)^{-n/2} \exp\left\{-\frac{1}{2\hat{b}_n^2}\sum_{i=1}^{n}(y_i - \hat{a}_n h(t_i))^2\right\}$$

and the likelihood ratio

$$R = \frac{f_{a_0,b_0^2}(y_1, \dots, y_n)}{f_{\hat{a}_n,\hat{b}_n^2}(y_1, \dots, y_n)} = \frac{(2\pi b_0^2)^{-n/2}\exp\{-(1/2b_0^2)\sum_{i=1}^{n}(y_i - a_0 h(t_i))^2\}}{(2\pi\hat{b}_n^2)^{-n/2}\exp\{-(1/2\hat{b}_n^2)\sum_{i=1}^{n}(y_i - \hat{a}_n h(t_i))^2\}}$$

$$= \left(\frac{\hat{b}_n^2}{b_0^2}\right)^{n/2} \exp\left\{\frac{1}{2}\left[n - \frac{1}{b_0^2}\sum_{i=1}^{n}(y_i - a_0 h(t_i))^2\right]\right\}$$