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Effects of triad formations stimulated by intermediaries on network topology

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HIGHLIGHTS

- We examine network models based on triad formation stimulated by existing nodes.
- A growth model where the triad is required for "birth" of a new vertex is proposed.
- Triad formation induces hubs from vertices with relatively larger degrees.
- Triad formation induces typical behaviors of local clustering strength.

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ABSTRACT

Triad formation of vertices is considered a significant mechanism in the emergence of highly clustered structures in real networks. However, the net effect of triad formations on network topology has yet to be understood completely, since triad formations are usually studied with additional effects including the attachment of new vertices to prevent a saturation of the number of edges, where almost all vertices are directly linked to each other. In this paper, we focus on the net effects of triad formations stimulated by randomly chosen intermediaries on network topologies such as local clustering and evolution of degrees. We show that the local clustering of vertices with degree k is divided into an essential term $\sim 1/k$ which can be widely seen in real networks and additional terms depending on the initial network topology. Also, we derive an equation which measures the influence of local structures of networks on the time evolution of vertex degrees, according to which triad formations lead to the so called "rich get richer" phenomenon in the evolution of degrees. Local events like a triad formation stimulated by pre-existing vertices leads not only to highly clustered structures but to a typical power-law form in the degree distribution with a power-law exponent of about 2.

1. Introduction

The study on the evolution process of large-scaled networks has attracted considerable attention from researchers, since common structures have been found in various networks including the Internet, biological networks, and social networks [1–4]. The most well-known structures in such studies are: the scale-free property which is the power-law in the vertex degree distribution [5], and the highly clustered structure [6] in which there is the high tendency for two friends of a person to also know each other, for example in the language of social networks. These topological structures are inseparable from

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studies on the functions of networks. For example, the scale-free property and locally clustered structure play important roles in the robustness of a network with respect to attacks or failures, epidemic processes [7], dynamical processes such as opinion formation, and various critical phenomena [8].

The modeling of networks is an effective tool to elucidate general principles in the formation of networks in the real world. For example, the growth property which allows the attachment of new vertices to the existing network gives a simple explanation for inequalities in the vertex degree [5]. However, only the growth property does not explain all the properties of the network structure, especially the clustered structure in real networks. Triad formation of vertices should be a presumable principle to provide a simple explanation for the emergence of highly clustered structure in networks. There are various versions of triad formation in the modeling of networks. For example, it is assumed that a new vertex is connected to both ends of a randomly or deterministically selected edge in Refs. [9–11], a new vertex is assumed to be able to connect simultaneously to a preferentially or randomly selected vertex and its neighboring vertices in Refs. [12–17], transformation of potential edges to normal edges is considered in Ref. [15], randomly chosen vertices stimulate two neighbors to create a triad relation of vertices in a network where random addition and deletion of vertices take place in Ref. [18], and triad formations along traces of random walkers are studied in Refs. [19,20]. These models considering triad formation have revealed mechanisms which reproduce local structures observed in real networks, such as the scaling property of the local clustering of vertices with degree k, $C(k) \sim k^{-\mathrm{a}}$ [21] where a is a constant which is typically about 1. However, most of these models assume multiple processes including the addition of new vertices which directly influence the evolution of degree distribution. This is because triad formations without attachment of new vertices lead to the saturation of the number of edges, where almost all vertices are directly linked to each other. This paper thus aims to focus on the net effects of the triad formation stimulated by randomly chosen intermediary on the network topology.

The triad formation is considered one local event where each vertex influences only its neighboring vertices. Such a local event without information on the entire network occurs commonly in our experiences such as in social networks. The accumulation of local events can affect not only the local structure of the network but also the global properties of networks such as degree distribution and the shortest path length between vertices, whereas typical mechanisms assumed in network modeling such as preferential attachment of vertices to existing vertices [5] and wiring of two vertices far from each other [6] explain mainly the global properties of networks. In real networks, there is a wide range of variations in the value of the power-law exponent in degree distribution and behavior of local clustering [21,22]. Such various properties should be generated by both global and local events. Not only models only considering global events such as wiring of two vertices far from each other but also other extreme models based only on local events are worth studying, in order to know to what extent the local event can reproduce various aspects of real networks.

The next section explains a model which considers the linkage of vertices stimulated by randomly chosen intermediaries in a random network with fixed number of vertices. Also discussed are theoretical predictions of clustering strength and relation between the rate of increase in degree, and local structure of random networks. In Section 3, numerical results are compared with theoretical predictions. In Section 4, the model is extended to a growth type in such a way that the rate of increase in degree follows the same principle as that in the previous model. The latter model is based on the idea that the intermediary not only introduces a pair of vertices but also urges them to form a triad relation with a new vertex. Therefore, the growth property in this model is also generated by local events. Numerical results for this network growth model are compared with theoretical predictions in Section 5. Section 6 summarizes the results and discusses the validity of theoretical predictions.

2. Effects of introductions by intermediaries

2.1. Model

Here we consider a network model where two vertices are introduced and joined by a common neighboring vertex. It should be noted that this model does not assume additions of new vertices. The network evolution is formulated by a transformation rule of a graph G_t with a subscript t as follows.

Initial condition Suppose that a random undirected graph G_0 is constructed as an initial graph by the following rule: First, consider isolated N vertices. Second, link a vertex i_1 to other m vertices randomly. Third, link randomly second vertex i_2 to other m vertices which have not been linked to i_2. Repeat this procedure from i_3 to i_N. As the result, a random undirected graph whose degree distribution is nearly Poissonian with the mean vertex degree $2m$ is produced. This paper deals with the case of $m = 2$.

Linking by introduction of an intermediary For each discrete time $t(\geq 1)$, a vertex in the graph is chosen randomly and it comes to serve as an intermediary between two neighboring vertices. The intermediary randomly chooses two vertices from its neighbors in graph G_{t-1}. If the chosen vertices have not been linked, link them. The renewed graph is represented by G_t which is unchanged from the previous graph G_{t-1} if the chosen vertices had been linked before time $t - 1$.

Iteration The addition of an edge according to the above rule is iterated.