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Social optimality in quantum Bayesian games

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HIGHLIGHTS

- Study game-theoretic solution concept of social optimality in a quantum Bayesian game.
- Our quantum Bayesian game uses the setting of generalized EPR experiments.
- A new stronger socially optimal outcome emerges in the quantum Bayesian game.

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ABSTRACT

A significant aspect of the study of quantum strategies is the exploration of the game-theoretic solution concept of the Nash equilibrium in relation to the quantization of a game. Pareto optimality is a refinement on the set of Nash equilibria. A refinement on the set of Pareto optimal outcomes is known as *social optimality* in which the sum of players' payoffs is maximized. This paper analyzes social optimality in a Bayesian game that uses the setting of generalized Einstein–Podolsky–Rosen experiments for its physical implementation. We show that for the quantum Bayesian game a direct connection appears between the violation of Bell's inequality and the social optimal outcome of the game and that it attains a superior socially optimal outcome.

1. Introduction

The probabilistic approach to the area of quantum games [1–64] shows how quantum probabilities can result in non-classical outcomes that defy intuition. Without referring directly to the mathematical machinery of quantum mechanics, this approach shows how a game-theoretic setting can benefit from quantum probabilities obtained from quantum mechanical experiments. As classical probabilities, quantum probabilities are normalized but they also obey other constraints that are dictated by the rules of quantum mechanics. Our earlier studies [58,59] have investigated coevolution [65] in quantum games and also developing a probabilistic approach to quantum games [24,48,50,60]. The solution concept of the Nash equilibrium (NE) from non-cooperative game theory [66–68] has been studied from the outset of the field of quantum game theory. Named after John Nash, it consists of a set of strategies, one for each player, such that no player is left with the incentive to unilaterally change her action. Players are in equilibrium if the change in strategies by any one of them would lead that player to earn less than if she remained with her current strategy.

For a physical system used in implementing a quantum game, one usually considers violation of Bell's inequality in relation to the outcome of the game as represented by the concept of a NE. In almost all cases of interest within the area of quantum game theory, one notices that Bell's inequality violation may only be indirectly proportional to the NE (or equilibria) of the game. The question then arises whether there exist other solution concepts for which the mentioned connection

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becomes a direct proportionality. In the present paper, it is shown that when one considers a particular refinement of the NE concept, a direct connection can indeed be established between Bell's inequality and the outcome of the game as represented by that refinement.

Pareto optimality is a refinement on the set of Nash equilibria. An outcome of a game is Pareto optimal if there is no other outcome that makes every player at least as well off and at least one player strictly better off. That is, a Pareto optimal outcome cannot be improved upon without disadvantaging at least one player. Quite often, a NE is not Pareto optimal implying that players' payoffs can all be increased. For the Prisoners' Dilemma game, it is well known that the NE is not Pareto optimal.

A stronger refinement of the NE concept, which is even simpler to state, is known as *social optimality* [69]. It is a choice of strategies, one by each player, which is a social welfare maximizer as it maximizes the sum of the players' payoffs. Of course, this definition is only appropriate to the extent that it makes sense to add the payoffs of different players together. Maximizing the sum of players' payoff may not necessarily lead to the satisfaction of all the participating players.

As the Pareto optimal outcome is a refinement on the set of NE, the outcomes that are socially optimal must also be Pareto optimal. If such an outcome were not Pareto optimal, there would be a different outcome in which all payoffs were at least as large, and one was larger. This would be an outcome with a larger sum of payoffs. On the other hand, a Pareto optimal outcome need not be socially optimal and a NE may not be socially optimal even though it can be Pareto optimal. However, a socially optimal outcome is always Pareto optimal.

In an earlier work [60] we have studied the solution concept of a Bayesian Nash equilibrium using a probabilistic approach to quantum games [24,48,50,60]. In the present paper, we study how quantum probabilities can result in a different outcome for the solution concept of social optimality in a Bayesian game. We notice that with this refinement of the NE concept, a direct connection appears between violation of CHSH form of Bell's inequality [70,77,78] and the social optimal outcome of the Bayesian game.

2. Social optimality in a Bayesian game

The decision-makers in a strategic game are called players who are endowed with a set of actions. In a Bayesian game [66–68] the players have incomplete information about the other players' payoffs i.e. the payoffs are not common knowledge. Incomplete information means that at least one player does not know someone else's payoffs. Random values are assigned to the players that take values of *types* for each player and probabilities are associated to those types. A player's payoff function is determined by his/her type and the probability associated with that type. Examples include auctions in which bidders do not know each others' valuations and bargaining processes in which another player's discount factor is unknown. The game of Battle of the Sexes [68] when one does not know if the other prefers to be alone or go on a date provides a clearer exposition of a social dilemma that can be analyzed using the theory of Bayesian games. More formally [67,68], a Bayesian game consists of players and states. Each player has a set of actions, a set of signals that the player may receive, and a signal function that associates a signal with each state. Consider the following Bayesian game in which there are four states represented by four rectangular boxes. As can be noted, each state is a complete description of one collection of the players' relevant characteristics. Players do not observe the state and instead receive signals that provide them information about the state. The signal functions [68] map states into players' types. For the game in Table 1, the players Alice and Bob both have two types that we call type 1 and type 2.

As Table 1 shows, for Alice of either type, Bob is found to be one of the two types with probability $\frac{1}{2}$. Similarly, for Bob of either type, Alice is found to be one of her two types with the probability $\frac{1}{2}$. The payoffs to Alice and Bob of each type are given in Table 1. For instance, from Table 1, the payoff matrix describing the interaction between Alice of type 2 and Bob of type 1 is given as

$$\begin{array}{cc} & \text{Bob of type 1} \\ & \begin{array}{cc} B & S \end{array} \\ \text{Alice of type 2} \begin{array}{c} B \\ S \end{array} & \begin{pmatrix} (1,1) & (0,0) \\ (0,0) & (1,1) \end{pmatrix} \end{array}. \tag{1}$$

The normal form representation of this Bayesian game is obtained in Table 2 where, the entries in parentheses on left column are pure strategies for Alice's two types, respectively. Similarly, the entries in parentheses in the top row are pure strategies for Bob's two types, respectively. For the two pairs of payoff entries, the first pair is for Alice's two types and the second payoff pair is for Bob's two types.

For instance, in Table 2 consider the entry $(\frac{1}{2}, 1), (\frac{1}{2}, 1)$ that corresponds when the strategy of Alice's two types is (S, B) and the strategy of Bob's two types is (B, S). Here, understandably, the first entry in the braces is the strategy of either player of type 1. Now we refer to Table 1 and notice how Table 2 is obtained from Table 1. At the intersection of the two rows representing the strategies of S, B, played by Alice's first and second types, respectively, and the two columns representing the strategies of S, B, played by Alice's first and second types, respectively, we find the following entries

$$\begin{array}{lcc} & \text{Bob's first type: } B & \text{Bob's second type: } S \\ \text{Alice's first type: } S & (0,0) & (1,1) \\ \text{Alice's second type: } B & (1,1) & (1,1) \end{array}, \tag{2}$$