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Quantum gravity corrections in Chandrasekhar limits

Mohamed Moussa

Physics Department, Faculty of Science, Benha University, Benha 13518, Egypt

HIGHLIGHTS

- Modification in the pressure of white dwarf due to quantum gravity is investigated.
- A modulation in Lane–Emden equation is performed due to the change in pressure.
- Mass and radius of the star is calculated with different quantum gravity parameter.
- Increase in the central density leads to a decrease in mass and radius of the star.
- Quantum gravity leads to decrease the mass limit and increased radius of the star.

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ABSTRACT

It is agreed that Chandrasekhar mass and central density of white dwarfs are independent, which means that there is a whole series of stars having radius and central density as parameters that all have the same Chandrasekhar mass. In this article the influence of a quantum gravity is shown so the Chandrasekhar limits (mass and radius) depend explicitly on the central density and gravity parameters. A new polytropic relation between degenerate pressure of the star and its density is investigated. This leads to a modification in Lane–Emden equation and mass and radius formulas of the star. A modified Lane–Emden equation is solved numerically with consideration to the mass density of the star depends on its radius. The solution was used in calculating the mass and radius limit of the white dwarf. It was found that mass and radius limits decrease due to increase in central density and gravity parameters in a comparison with the original values. We can say that central density and quantum gravity constitute a new tool that can help to make the theoretical values corresponding to experimental observations apply in a better manner.

1. Introduction

The most important prediction of various quantum theories of gravity is the existence of minimum measurable length and a maximum measurable momentum near the Planck scale, and hence the continuum picture of spacetime breaks down. The black holes physics indicates that the minimum length is of order of the Planck length which should act as a universal feature of all models of quantum gravity Refs. [1,2]. Also, in the context of perturbative string theory, the strings are the smallest probe available, and so, it is not possible to probe the spacetime below the string length scale. Thus the string length scale acts as a minimum measurable length in string theory Refs. [3–7]. This makes generalization process of uncertainty principle a significant request in physics. The results is a presence of a deformed or generalized uncertainty principle (GUP) which is equivalent to a modification in the commutation relations between position coordinates and momenta (deformed Heisenberg algebra) Refs. [6–15]. It is known that the familiar uncertainty relation is closely related to the canonical Heisenberg algebra, this way the modified canonical Heisenberg algebra is related to a non-canonical one.

E-mail addresses: mohamed.ibrahim@fsc.bu.edu.eg, moussa.km@gmail.com.

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So that the commutator of the position and momentum operators becomes momentum dependent, instead of a constant. With this non-canonical algebra, the coordinate representation of the momentum operator get modified, and this in turn produces correction terms for all quantum mechanical systems. On the other hand, Doubly Special Relativity (DSR), another approach to quantum gravity, leads to a deformation in Heisenberg algebra, [16–18]. It inspires both the velocity of light and Planck energy as universal constants. The deformed Heisenberg algebra studied in DSR theory has been predicted from many consequences, such as, discrete spacetime Ref. [19], spontaneous symmetry breaking of Lorentz invariance in string field theory Ref. [18], spacetime foam models Ref. [20], spin-network in loop quantum gravity Ref. [21], non-commutative geometry Ref. [22], and Hoȓava–Lifschitz gravity Ref. [23]. Another approach to quantum gravity through a modification in dispersion relation, this condition implies a breakdown of Lorentz symmetry Refs. [20,24,25]. This model was tested extensively in physics, for example Refs. [26–29].

Ali, Das and Vagenas Refs. [30,31] worked on a new approach for quantum gravity. They suggested commutators that are consistent with string theory, black hole physics and DSR and ensure $\left[x_i, x_j\right] = \left[p_i, p_j\right] = 0$ (via Jacobi identity). The new commutator have the following form

$$\left[x_i, p_j\right] = i\hbar\left[\delta_{ij} - \alpha\left(p\alpha_{ij} + \frac{p_i p_j}{p}\right) + \alpha^2(p^2\delta_{ij} + 3p_i p_j)\right] \tag{1}$$

where $\alpha = \frac{\alpha_0}{M_P c} = \frac{\alpha_0 l_P}{\hbar}$, where M_P, l_P and $M_P c^2$ are Planck mass, length and energy, respectively. This in turn imply a minimum measurable length and a maximum measurable momentum in such a way $\Delta x \geq (\Delta x)_{\text{min}} \approx \alpha_0\, l_P$ and $\Delta p \leq (\Delta p)_{\text{max}} \approx \frac{M_P\, c}{\alpha_0}$. As a result, according to Eq. (1), GUP modifies the physical momentum Refs. [30–32]

$$x_i = x_{0i}, \qquad p_i = p_{0i}\left(1 - \alpha p_0 + 2\alpha^2 p_0^2\right) \tag{2}$$

where x_{0i} and p_{0i} satisfy the canonical commutation relation $[x_{0i}, p_{0i}] = i\hbar\delta_{ij}$, so we can consider p_i as a momentum in Planck scale and p_{0i} as a momentum at low energies (having standard representation in position space $p_{0i} = -i\hbar\frac{\partial}{\partial x_{0i}}$). It is assumed that the dimensionless parameter α_0 is of the order of unity, in which case the α dependent terms are important when energies (momenta) are comparable to Planck energy (momenta) and length is comparable to the Planck length Ref. [30]. The upper bounds on the GUP parameter α have been derived in a lot of previous works for example Refs. [32–38].

On the other hand the current observation indicates that the white dwarf has smaller radius than the theoretical predictions. This leads us to introduce quantum gravity as a tool to explain this defect. It is worth mentioning that this problem is considered with many quantum gravity approaches. In Ref. [39] Camacho assume a constant density of white dwarf and he calculated the star radius with the modified dispersion relation that caused by a breakdown in Lorentz symmetry. It is found that the change in the star radius depends on the sign of the quantum gravity parameter. Amelina-Camellia et al. Ref. [40] try to improve these results using a small modification in density of state by assuming that the law of composition of momenta affects the rules of integration over energy–momentum space and these are crucially relevant for Chandrasekhar analysis. The authors in Ref. [41] extend the analysis of Camacho by stopping the unphysical assumption of constant density. They assumed that the density is not constant throughout the star and reported the numerical solutions to the exact equations for the Chandrasekhar. The result is the realistic density that shows significant corrections at Planck scale and the mass limit is raised or lowered according to the sign of the modification. The stability of white dwarfs is also examined using a non-commutative geometry concept in Ref. [42]. Another approach of generalized uncertainty principle is contracted, concepts of this approach is reported Refs. [7,14,15,43]. Using this approach the author in Refs. [44–46] found that quantum gravity correction depends on the number density of the star and it tends to resist the star collapse.

We will consider Chandrasekhar limit with this quantum gravity approach. The analysis in Refs. [47,48] will be extended by disregarding the unphysical assumption of constant density. In fact pressure and density of the star depend on the star radius. The modified star pressure will be calculated which leads to a modification in Lane–Emden equation. This equation will be solved numerically in order to determine the quantum gravity corrections on the mass and radius of the star.

2. Modified pressure inside white dwarfs

In order to investigate quantum gravity effects the statistical mechanical equations should be put in a form consistent with GUP framework. The GUP can be considered in phase space analysis by two equivalent ways. First considering deformed commutator with non-deformed Hamiltonian function (i.e. deformed the measure of integration). Second calculating canonical variables on the GUP corrected phase space which satisfy the standard commutative algebra (i.e. non-deformed standard measure of integration), in this case the Hamiltonian function should be deformed. These two pictures are related to each other by Darboux theorem. In this work we will consider a deformed measure of integration with non-deformed Hamiltonian function. In order to do that the effect of GUP on the density of states should be considered such that the number of microstates inside the phase space volume does not change with time, this what is called Liouville theorem [49,4]. To do that let us begin with the Heisenberg equations of motion

$$\dot{x} = \frac{1}{i\hbar}[x, H], \qquad \dot{p} = \frac{1}{i\hbar}[p, H]. \tag{3}$$