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Evolution to the equilibrium in a dissipative and time dependent billiard

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ABSTRACT

We study the convergence towards the equilibrium for a dissipative and stochastic time-dependent oval billiard. The dynamics of the system is described by using a generic four dimensional nonlinear map for the variables: the angular position of the particle, the angle formed by the trajectory of the particle with the tangent line at the position of the collision, the absolute velocity of the particle, and the instant of the hit with the boundary. The dynamics of the stationary state as well as the dynamical evolution towards the equilibrium is made by using an ensemble of non interacting particles. Finally, we make a connection with the thermodynamic by using the energy equipartition theorem.

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1. Introduction

Classical billiards are dynamical system in which a particle, or an ensemble of non-interacting particles, moves confined to and experiences collisions with a boundary [1–9]. Basically, they are settled in three classes, namely (i) integrable, (ii) ergodic, and (iii) mixed. In case (i), the phase space consists of invariant tori filling the entire phase space. In case (ii), the time evolution of a single initial condition is enough to fills up the entire phase space. Finally, in case (iii), one can observe invariant tori, chaotic seas generally surrounding Kolmogorov–Arnold–Moser (KAM) coexisting. If a time dependent perturbation is introduced on the boundary [10], the system exchanges energy with the moving particles upon collisions. Such type of systems have attracted a lot of attention lately because they can be used to study the phenomenon of unlimited energy growth also known as Fermi acceleration [11]. However, how to identify in which type of system the phenomenon of Fermi acceleration will be observed? To answer this question, Loskutov, Ryabov and Akinshin (LRA) proposed a conjecture where they state that a chaotic component in the phase space for the time-independent dynamics is a sufficient condition to observe Fermi acceleration once a time dependent perturbation on the boundary is introduced. This conjecture became known as LRA conjecture [12,13] and over the years it has been verified in several systems such as the time dependent Lorentz gas [14,15], oval [16] and stadium [17,18] billiard among many other systems [19,20]. Nevertheless, later on results have shown that the existence of a chaotic component is a sufficient, but not a necessary condition for Fermi acceleration since the unlimited energy growth was also observed in a time dependent elliptical billiard. As it is known, the elliptical with static boundary is an integrable system whose integrability comes from the conservation of the angular momentum with

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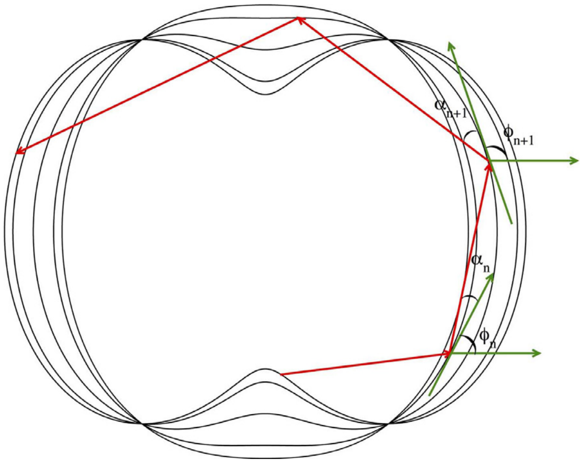

Fig. 1. Illustration of 5 snapshots of a time-dependent oval billiard. The corresponding angles that describe the dynamics of the model are also shown for two collisions.

respect to the two foci [21]. However, when a time dependent perturbation is introduced into the system, the separatrix is replaced by a chaotic layer and trajectories that were confined inside the separatrix (librators) can now explore the region outside the separatrix (rotators) and vice-versa. This change of behavior, namely, librator orbits "jumping" to rotator and vice-versa turned out to be the mechanism which produces the unlimited energy growth [22–24]. More recently, it became clear that such a phenomenon is not robust [25] since a tiny amount of dissipation, either upon collision [26,27] or during the flight [28], is enough to suppress Fermi acceleration.

The motion of the time dependent boundaries can be related to a more physical situation. Indeed due to the thermal fluctuations, the position of each atom on the boundary is allowed to move locally. Such oscillation of the atoms, and hence of the boundary, can be extended to the context of billiard which allows us to connect the observables obtained from the velocity of the particle – such as the kinetic energy – to the thermodynamics, more precisely, the temperature and entropy [29]. So far, such a connection has been made for the Lorentz gas to describe the motion of electrons between heavy ions as in a lattice of metal [30]. Therefore, further investigation is needed to understand how the dynamics of the systems can be connected to more real situations.

In this paper we study some dynamical properties for an ensemble of noninteracting particles in a time dependent billiard. Our main goal is to understand and describe the dynamics of the mean squared velocity as a function of the time considering different values of the control parameters, namely, the dissipation upon collision, the parameter that controls the shape of the boundary and the amplitude of the time-dependent perturbation of particles moving inside a closed domain. As an illustration, we consider a stochastic version of the time-dependent oval billiard. The introduction of a random perturbation on the border resembles the rough oscillations – producing a random exchange of energy upon collision – at least in the microscopic domain. The results that are obtained numerically are confirmed by using an analytical approach.

This paper is organized as follows. In Section 2 we present the model. In Section 3 we present our numerical findings. In Section 4 we introduce an analytical approach to obtain the behavior of the average velocity as a function of the number of collisions with the moving boundary. Finally, in Section 5, we make a connection with the thermodynamics by using the expression obtained analytically for the particle's average velocity. Conclusions are drawn in Section 6.

2. The model and the map

In this section we present all the details needed to study the dynamics of an ensemble of noninteracting particles experiencing collisions with a moving boundary. As it is so usual in the literature, we describe the dynamics of the model in terms of a four dimensional nonlinear mapping $\tilde{T}$ that gives the angular position of the particle θ_n; the angle that the trajectory of the particle forms with the tangent line at the position of the collision α_n; the absolute velocity of the particle $|\vec{V}_n|$ and the instant of the hit with the boundary t_n, i.e., $(\theta_{n+1}, \alpha_{n+1}, |\vec{V}_{n+1}|, t_{n+1}) = \tilde{T}(\theta_n, \alpha_n, |\vec{V}_n|, t_n)$. The index n denotes the nth collision with the moving boundary. Assuming that the shape of the boundary in polar coordinates is $R_b(\theta, \epsilon, p, t) = 1 + \varepsilon f(t)\cos(p\theta)$ where the subindex b denotes boundary, $f(t)$ is a function to be chosen, ε is the perturbation of the circular billiard and p is an integer number warranting a closed boundary, otherwise particles would escape. We consider $f(t) = 1 + \eta\cos(\omega t)$ where η is the amplitude of the time dependent perturbation and ω is the angular frequency, which from now is fixed as $\omega = 1$. Fig. 1 shows a typical illustration of a billiard and the angles used to describe the dynamics of the model.

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