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Navigability of multiplex temporal network

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HIGHLIGHTS

- We consider the dynamics of three walk processes and the multiplex temporal network at the same time scale.
- The multiplex structure with the different interconnection strengths D may change the nodes importance to some extent.
- The strategies of RWC and RWP speed up the less diffusive of the layers.
- The strategy of RWE enhances the diffusion of both layers.

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ABSTRACT

Real world complex systems have multiple levels of relationships and in many cases, they need to be modeled as multiplex networks where the same nodes can interact with each other in different layers, such as social networks. However, social relationships only appear at prescribed times so the temporal structures of edge activations can also affect the dynamical processes located above them. To consider both factors are simultaneously, we introduce multiplex temporal networks and propose three different walk strategies to investigate the concurrent dynamics of random walks and the temporal structure of multiplex networks. Thus, we derive analytical results for the multiplex centrality and coverage function in multiplex temporal networks. By comparing them with the numerical results, we show how the underlying topology of the layers and the walk strategy affect the efficiency when exploring the networks. In particular, the most interesting result is the emergence of a super-diffusion process, where the time scale of the multiplex is faster than that of both layers acting separately.

1. Introduction

Random walks represent real diffusive processes in networks that are better described by a finite number of discrete visits. This process is at the core of many real-world dynamical processes ranging from physics to other disciplines, such as financial time series analysis [1], genetics [2], social sciences [3], contagion processes [4,5], and the ranking of websites on the world wide web [6] based on importance and quality. Recent studies [7–9] have provided some effective walk strategies for navigating and exploring complex networks where the structures of the networks are monoplex. However, many real world networked systems are coupled together via a complex pattern of interdependencies. Thus, they can be described as multiplex networks where the same nodes can interact with each other in different layers to form a set of interacting and co-evolving networks. Examples of these multiplex systems include social [10] and transportation networks [11], neural and

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brain networks [12], and technological networks [13]. Moreover, the edges that represent the interactions in a social network may also vary over time [14,15], i.e., they appear only at prescribed times. The edge's temporal structure can also affect the dynamics of systems by interacting through the network [16–21] if we still project them onto a weighted monoplex network, so some information will be lost. Thus, investigating the dynamics of networks with multiplex and temporal properties is an essential and challenging task to uncover the nature and full breadth of social interactions.

In order to investigate how the temporal structure of a network affects the random walk processes that occur within it, Perra et al. [22] used a graph sequence framework to develop an activity-driven model of temporal networks. Subsequently, the dynamics of the unbiased random walk and temporal network were considered at the same time scale [18], which detected strikingly different behavior compared with that over static topologies. Thus, Wang [23] considered a biased random walk by introducing a special edge-weighting form and provided a more effective search strategy. Recently, Moinet et al. [24] analyzed the burstiness and aging in a time-integrated network by introducing a power-law form of the waiting time distribution of consecutive interactions. To investigate how the multiplex structure affects the random walk process, another study [25] showed that diffusion processes in multiplex networks can exhibit enhanced-diffusive behavior, where this phenomenon is strictly related to the setup of the particular multiplex, and it has no counterpart in classical monoplex networks. Subsequently, numerical results [26] proved that the structures of multiplex networks are more robust to random failures than their individual layers. These findings are important for the future development of search and navigation strategies in real temporal-interconnected systems. In the present study, by combining the temporal and multiplex properties of a real system, we investigate three different random walk processes on multiplex temporal networks. We analytically estimate the multiplex centrality and coverage $C(t)$ (defined as the number of different nodes visited by the walker at time t) for three types of walk processes, as well as demonstrating how the underlying topology of the layers and the walk strategies affect the efficiency when exploring networks.

The remainder of this paper is organized as follows. In Section 2, we modify the time-varying network presented previously [22] to obtain a model of a multiplex temporal network. In Section 3, we describe three representative random walk processes according to their different transition rules, and we define their steady-state behaviors in terms of the activities of the nodes in two layers and the interconnection strength. In Section 4, we estimate the coverage function and show how the efficiency when exploring multiplex temporal networks depends critically on the strengths of their interconnections and the specific walk strategy. Finally, we summarize our results and make some suggestions for future research in Section 5.

2. Multiplex temporal networks

As described previously [22], we assume that G_t denotes a simple graph at (discrete) time t and we introduce inter-layer connectivity to extend the model to a multiplex temporal network, which has two layers and each layer is the same size N but has different connected patterns. Initially, these $2N$ nodes are disconnected and they can be updated in random order at each iteration step. In one iteration step, a node i in layer 1 may become active with a probability of a_i, but its counterpart node $N+i$ in layer 2 is independently active with a probability of b_i, where a_i, b_i are both bounded in the interval $(\epsilon, 1)$ and they are extracted from the power-law distributions of $F(a) \propto a^{-3}$ and $F(b) \propto b^{-2}$, respectively. Thus, there is no correlation between the activities of the nodes and their counterparts. Then, a multiplex temporal network process is defined according to the following rules.

(1) Increase the time counter to t and let the instantaneous multiplex network G_t start with $2N$ disconnected nodes.

(2) For every node i in layer 1, make it active with a probability of $a_i \Delta t$. Connect i to $N+i$ and randomly choose m other distinct nodes (active or not) in the same layer to generate neighbor links.

(3) For every node $N+i$ in layer 2, make it active with a probability of $b_i \Delta t$. Connect $N+i$ to i and randomly choose m other distinct nodes (active or not) in the same layer to generate neighbor links.

(4) At the next time step $t+\Delta t$, all the edges in the network G_t are deleted and the process starts again to generate the network $G_{t+\Delta t}$.

When a node i (i is layer 1 or layer 2) is active, the $m+1$ generated links can be outgoing edges, and thus the multiplex temporal network is actually a directed network.

3. Random walk on a multiplex network

3.1. Three random walk processes on a multiplex network

We define the random walk process on multiplex temporal networks as follows. At each time step t, if the instantaneous multiplex network G_t is generated according to the network generation process and for each unit time (setting $\Delta t = 1$), the walker located on a node can hop to a node within the same layer as well as a node in a different layer. For the diffusion process on layer 1, we consider the same unbiased walk strategy described previously [18], and we choose the optimal exploration method for layer 2 based on a biased random walk (assuming that the tunable parameter θ is 1.0). After diffusion, a new instantaneous network G_{t+1} is generated at time $t+1$ and the walker also diffuses again for a time. Thus, we investigate the concurrent dynamics of the random walker and the network that occur over the same time scale. The walker can become