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Quantum kinetics of spinning neutral particles: General theory and Spin wave dispersion

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HIGHLIGHTS

- Kinetic equations derived in terms of many-particle quantum hydrodynamic method.
- The long-range spin–spin interaction is the main mechanism of wave propagation.
- Dispersion and collisionless damping of spin waves are calculated.
- Dispersion of three- and two-dimensional spin waves is calculated.

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ABSTRACT

Plasma physics gives an example of physical system of particles with the long-range interaction. At the small velocities of particles we can consider the plasmas approximately as the systems of particles with the Coulomb interaction. The Coulomb interaction is an isotropic interaction. Systems of spinning neutral particles are involved in the spin–spin interaction, which is a long-range anisotropic interparticle interaction. Therefore they can reveal more rich properties than the Coulomb plasmas. Furthermore, to study the systems of spinning particles we can develop the kinetic and hydrodynamic methods similar to the methods applying in the plasma physics. We derive the kinetic equations by a new method, which is the generalization of the many-particle quantum hydrodynamics. Obtained set of the kinetic equations is truncated, so we have a closed set of two equations. First of them is the kinetic equation for the quantum distribution function. The second equation is the equation for the spin-distribution function, which describes the spin kinetic evolution and contributes to the time evolution of the distribution function. Our method allows one to obtain equations for both the three dimensional systems of particles and the low dimensional systems. Hence we consider the spin waves in the three- and the two-dimensional systems of neutral spinning particles.

1. Introduction

The study of quantum plasmas involves the consideration of the spin evolution and its influence on the properties of degenerate plasmas. Corresponding hydrodynamic and kinetic equations are required for the quantum plasma study. The method of the rigorous derivation of the quantum hydrodynamic equations from the many-particle Schrödinger (Pauli) equation was suggested by Kuz'menkov and Maksimov in 1999–2001 [1–3]. Spin evolution and its contribution in the set of quantum hydrodynamic equations were considered there along with the exchange interactions. General form of the contribution of the exchange interaction in the quantum hydrodynamic equations for bosons and fermions was

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demonstrated in these papers. These general properties were also used for the Bose–Einstein condensates and degenerate Fermi gases of neutral atoms [4], where exchange interaction plays the main role. The Wigner quantum kinetics was also used for the quantum plasma description and obtaining the corresponding quantum hydrodynamic equations [5–9]. The Wigner quantum kinetics allows one to consider the Coulomb exchange interaction in three-dimensional [5,6], and two-dimensional electron gas [7]. Generalization of results of Refs. [5–7] were recently obtained by the many-particle QHD method [10]. Many papers on the quantum plasmas have been published. Some results were discussed in reviews [11–13]. In this paper we suggest a new method of the quantum kinetic equations derivation. We apply the derived equations to study the waves in the systems of neutral spinning particles, which give an example of the anisotropic long-range interaction. In these systems the Coulomb interaction does not obscure the spin–spin interaction.

We present a method of the quantum kinetic equation derivation. This method is the direct generalization of the basic ideas of the many-particle quantum hydrodynamics suggested by L.S. Kuz'menkov and S.G. Maksimov [1,2]. We use the operator of the quantum distribution function of N particles, which corresponds to the classical one. Our definition of the quantum kinetic function also corresponds to the explicit definition of the particle concentration, which one of the main definitions in the many-particle quantum hydrodynamics [1]. Considering spinning particles with the spin–spin interaction we come to a set of two kinetic equations, one for the distribution function, and another one for the spin distribution function. Both these functions have three arguments $f = f(\mathbf{r}, \mathbf{p}, t)$ and $\mathbf{S} = \mathbf{S}(\mathbf{r}, \mathbf{p}, t)$. The presented theory is suitable for different physical systems, as it was demonstrated in Refs. [14,15]. First of all it can be applied to the quantum plasma, where one studies the evolution of charged spinning particles. However we focus our attention on the neutral spinning particles to study the spin waves in the magnetized dielectrics. A distribution function depending on four arguments has been considered in literature [16–18]. This distribution function has the following arguments: time, space coordinates, momentum, and spin. In this case, a dependence of an equilibrium distribution function on spin has been considered in Ref. [17, see formula (2)].

A lot of achievements in the quantum kinetics are based on the Wigner's theory [19]. However there are other methods for the quantum kinetics along with the Wigner's treatment [14,20–23].

The Wigner kinetics has allowed to derive a set of kinetic equations for spinning particles [24] having the same structure as we obtain in this paper by means of generalization of the many-particle quantum hydrodynamic method.

The Wigner kinetics has been applied to consider the relativistic plasmas of particles governed by the Dirac [25,26] and the Klein–Gordon [27] equations. Along with the recent works, it is worthwhile to mention earlier papers on semi-relativistic kinetics of spinning particles [28–31]. The Wigner kinetic finds its applications in derivation of the quantum kinetic equations, which govern the evolution of neutrino flavor at high density and temperature [32]. The electron quantum kinetics is analyzed with the help of the electric current and transferred angular momentum in Ref. [33]. The equivalent of the Stosszahlansatz of classical statistical mechanics is derived entirely on the basis of quantum field theory for a model system, without invoking any of the common extra-mathematical notions of particle trajectories, collapse of the wave function, measurement, or intrinsically stochastic processes [34].

The quantum kinetics obtained in terms of the retarded and more or less nonequilibrium Greens functions in the mixed Wigner coordinates finds a lot of applications in recent research of different physical systems (see for instance [35–39]). Discussion of some methods of derivation of the kinetic equations were recently discussed in Ref. [40].

It is well-known that the Pauli equation for one charged spinning particle in an external electromagnetic field can be presented in the form of hydrodynamic equations [41]. They are the continuity, the Euler (equation of the velocity field evolution), and the Bloch (equation for the spin density evolution) equations. The Euler equation for one particle contains the density of force describing action of the external field on the particle. Instead of the thermal pressure one can find the quantum pressure, which corresponds to well-known quantum Bohm potential. It contains contribution of the spin density along with the spin independent part. The Bloch equation was suggested for the classical magnetic moment. It shows the evolution of the magnetic moment in the external magnetic field. Hydrodynamical Bloch equation appears for the vector field of the spin density. This equation contains divergence of the spin–current and the torque field caused by the external field. One-particle spin–current consists of the two parts: the kinematic term, which is the product of the spin density on the velocity field, and the quantum term, which is an analog of the quantum Bohm potential in the Euler equation.

The fact that the one-particle Pauli equation can be presented as the described set of the quantum hydrodynamic equations leads to some applications. Spinless part of the hydrodynamic equations looks like the classic hydrodynamic equations for the systems of charged particles: the classic plasma. Technically, there are only two differences between these sets. The first difference is the presence of the quantum Bohm potential instead of the thermal pressure. The second difference is an explicit form of the force density appearing as the Lorentz force. However the set of quantum equations obtained from the one-particle Pauli equation contains the force caused by the external fields only. Whereas, in the classical hydrodynamics, for the many-particle systems, we have the force of interparticle interaction along with the external force. Internal fields describing the interparticle interaction satisfy the Maxwell equations, where the charge and current densities are given by the particle evolution.

One can use this similarity of the one-particle quantum hydrodynamics and the hydrodynamic equations of classic plasma [42,43]. To this end one should account for the thermal pressure along with the quantum Bohm potential and assume that he has full force density coupled by the Maxwell equations. Thus, the set of quantum hydrodynamic equations can be used to estimate the contribution of the spin evolution in the properties of quantum plasmas. Nevertheless this manipulation is not necessary. We have mentioned above that the rigorous derivation of the many-particle quantum hydrodynamics