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Effects of periodic force on the stability of the metastable state in logistic system☆

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HIGHLIGHTS

- Effects of periodic force on the stability of the metastable state in logistic system is investigated.
- When the periodic force takes value of the first half cycle, the stability is weakened by the additive noise.
- When the periodic force takes value of the second half cycle, the stability is enhanced by the additive noise.
- The stability of the metastable state can be controlled with the value of periodic force.

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ABSTRACT

The effects of periodic force on the stability of the metastable state in logistic model are investigated. The expression of the mean first-passage time (MFPT) from the metastable state to the stable state is derived. Based on the expression, the effects of periodic force on the MFPT were analyzed. The results indicate that: (i) For the case of the multiplicative noise induced transition, the multiplicative noise and the periodic force weaken the stability of the metastable state; (ii) For the case of the additive noise induced transition, the stability of the metastable state is weakened by the additive noise when the periodic force takes value of the first half cycle, while the stability of the metastable state is enhanced by the additive noise when the periodic force takes value of the second half cycle; (iii) For the case of the correlation between the multiplicative and the additive noise induced transition, the correlation between the multiplicative and the additive noises and the periodic force weaken the stability of the metastable state.

1. Introduction

The logistic model is one of the most popular model not only in the mathematical ecology, but also in other applications of the cell growth and genetics, population dynamics etc. In recent years, it has been widespread concerned and researched [1–6]. The stationary statistics properties of the logistic system with cross white correlated noise was investigated [1]. The steady-state and transient statistical properties of the system in the case of cross correlated with the presence of correlated time are also investigated [2]. The stochastic resonance phenomenon in the system was also found [3–5]. The correlation function and the relaxation time of the system are computed and discussed [6].

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It has been revealed theoretically and experimentally that the average escape time from a metastable state in fluctuating or static potentials, as a function of the noise intensity, has non-monotonic behavior with the presence of a maximum value [7,8]. This means that the stability of metastable states can be enhanced by noise, i.e. the system remains in the metastable state for a longer time than in the absence of noise, this is called the noise-enhanced stability (NES) phenomenon [9]. Several investigations show that time delay and noise interaction can also induce noise enhanced stability phenomenon in different stochastic systems [10–14]. For the logistic system, we found that the stability of metastable state can be enhanced by increasing the cross-correlation degree between multiplicative noise and additive noise [15] and by increasing self-correlation time of multiplicative noise [16]. People want to prolong metastable life in actual system by adjusting the noise parameter, however, the regulation of noise parameter in actual system, which can only be achieved in a few systems, most systems or even impossible, because the noise is a random force, therefore, is very difficult to adjust it. Study on stochastic dynamics also show that some noise effect can be played by periodic force [17,18]. Then, the NES effect of stochastic systems, whether can also be played by periodic force? Namely, whether the stability of metastable in stochastic systems can be enhanced by the periodic force? This is a very worth exploring problem, because the periodic force is very easy to control. When tumor cells are under radiotherapy and a periodic chemotherapeutic treatment, role of periodic force should be considered for the system. If the periodic force can be also enhanced stability of the metastable state, it may realize in many actual systems to enhance stability of the metastable state in system. So we will carry out research on this problem in the logistic system.

The paper is organized as follows. In Section 2, the mean first passage time (MFFT) form metastable state to stable state is derived. In Section 3, the effects of the periodic force on the MFPT are analyzed. We end with conclusions in Section 4.

2. The mean first passage time of the logistic model with periodic force

The logistic model of the tumor cell growth is written as

$$\frac{dx}{dt} = ax - bx^2, \tag{1}$$

where x denotes cell population, a is the cell growth rate and b is the cell decay rate. This is an ideal equation without any fluctuation. By considering both internal and external fluctuations of the system, the Langevin equation reads [1,2]

$$\frac{dx}{dt} = ax - bx^2 + x\epsilon(t) - \Gamma(t). \tag{2}$$

When tumor cells are under radiotherapy and a periodic chemotherapeutic treatment, the growth rate of tumor cells should have a periodic form, for example, a sinusoidal form. If considering this environmental fluctuation, a term $A \sin \Omega t$ should be added to system, so Eq. (2) can be rewritten as [3]

$$\frac{dx}{dt} = ax - bx^2 + x\epsilon(t) - \Gamma(t) + A \sin \Omega t, \tag{3}$$

where $\epsilon(t)$ and $\Gamma(t)$ are Gaussian noises with zero mean, and

$$\langle \epsilon(t)\epsilon(t')\rangle = 2D\delta(t - t'), \tag{4}$$

$$\langle \Gamma(t)\Gamma(t')\rangle = 2\alpha\delta(t - t'), \tag{5}$$

and

$$\langle \epsilon(t)\Gamma(t')\rangle = 2\lambda\sqrt{D\alpha}\,\delta(t - t') \tag{6}$$

where α and D are the additive and the multiplicative noise intensities respectively. λ denotes the degree of correlation between $\epsilon(t)$ and $\Gamma(t)$ with $0 \le \lambda \le 1$. A and Ω is amplitude and frequency of input periodic signal respectively.

The deterministic potential corresponding to Eq. (3) reads

$$V(x) = -\frac{a}{2}x^2 + \frac{b}{3}x^3. \tag{7}$$

Fig. 1 shows that the deterministic potential respectively has one stable state corresponding to $x_s = \frac{a}{b}$ and one metastable state corresponding to $x_u = 0$. The Fokker–Planck equation corresponding to Eq. (3) can be obtained from Eqs. (3)–(6) [3]

$$\frac{\partial}{\partial t}P(x, t) = -\frac{\partial}{\partial x}B(x)P(x, t) + \frac{\partial^2}{\partial x^2}C(x)P(x, t) \tag{8}$$

where

$$\begin{aligned} B(x) &= ax - bx^2 + Dx - \lambda\sqrt{D\alpha}x + A \sin \Omega t, \\ C(x) &= Dx^2 - 2\lambda\sqrt{D\alpha}x + \alpha. \end{aligned} \tag{9}$$