Physica A 432 (2015) 102–107

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Global evidence on the distribution of firm growth rates[☆]

Michael A. Williams, Brijesh P. Pinto[*], David Park

Competition Economics LLC, 2000 Powell Street, Suite 510, Emeryville, CA 94608, United States

HIGHLIGHTS

- We use global data to examine empirically the distribution of firm growth rates.
- We use the data to test eight theoretical distributions with EDF statistics.
- The consensus finding in the literature supports the Laplace distribution.
- We find firm growth rates are best fit by the heavier-tailed Cauchy distribution.

ARTICLE INFO

Article history:
Received 1 October 2014
Received in revised form 27 January 2015
Available online 9 March 2015

Keywords:
Firm growth rates
Theoretical distributions
Empirical distribution function tests

ABSTRACT

The consensus finding in the literature is that the distribution of firm growth rates is best approximated by the Laplace distribution, a particular case of the Subbotin, or exponential power, family of probability distributions. Using a richer database than prior studies and testing for more theoretical distributions, we find that the distribution of firm growth rates is best approximated by the heavier-tailed Cauchy distribution.

1. Introduction

Starting with the path-breaking research of Stanley et al. [1], a consensus finding has emerged that firm growth rates display a "tent-shaped" Laplace distribution.[1] This result has been obtained in several studies—for example, Nunes Amaral et al. [5], Bottazzi et al. [6], Bottazzi and Secchi [7], Buldyrev et al. [8], Alfarano and Milaković [9], Riccaboni et al. [10], Hölzl [11], and Erlingsson et al. [12]. We explore the distribution of firm growth rates using a richer database than used in these prior studies. Our global database has 13,342 firms in 57 industries from 43 countries over the period 1999–2010. These firms had total 2010 revenues of $38.5 trillion or 61% of world GDP.

Building on prior work on the distribution of firm growth rates, we present empirical distribution function, or EDF, tests for the log of the sales growth rate for eight distributions: Cauchy, exponential, gamma, Laplace, logistic, log-normal, normal, and Weibull. We find, in common with the consensus finding, that the Laplace distribution performs reasonably well in approximating the distribution of firm growth rates. However, we show that the distribution of firm growth rates is

☆ We thank Leslie Park for research assistance. We thank Christophe Dutang, Hugo Mialon, Simon Wilkie, and Wei Zhao for helpful comments.

* Corresponding author.
E-mail addresses: mwilliams@competitioneconomics.com (M.A. Williams), bpinto@competitioneconomics.com (B.P. Pinto), dpark@competitioneconomics.com (D. Park).

[1] A related "scaling law" is one that pertains to firm size. For example, Axtell [2] finds that firm size is characterized by the Zipf distribution. Palestrini [3] studies the theoretical relationship between the Laplace firm growth-rates distribution and the Pareto firm size distribution. A related study by Cabral and Mata [4] demonstrates, using data on Portuguese manufacturing firms, that firm size distribution has evolved toward a lognormal distribution.

http://dx.doi.org/10.1016/j.physa.2015.02.103

substantially better approximated by the heavier-tailed Cauchy distribution, a particular case of the Lévy α-stable family of densities.[2] Thus, our results agree, for example, with the empirical finding of Bottazzi et al. [14] that "[w]hilst the Laplace distribution of growth rates was repeatedly found in previous studies and appeared to be emerging as something of a 'stylized fact', we observe here that the growth rates of French firms are even fatter tailed than expected, a property which holds with disaggregation".

Understanding the empirical distribution of firm growth rates is important because the distribution places constraints on theoretical models of firm growth. In turn, models of firm growth are generally based on dynamic models of innovation and investment (see, for example, Refs. [8,6]).[3] In order to be empirically relevant, such models should generate firm growth rate density functions consistent with the empirical densities. To date, no theoretical models exist in which firm growth rates follow a Cauchy distribution. Thus, our results invite research on dynamic models of competition in which firms' growth rates follow a Cauchy distribution.

2. Data and calculation of firm growth rates

We obtain data on firms' growth rates using a unique, proprietary database that provides financial data for 13,342 firms spanning 57 different industries across 43 countries for the years 1999–2010.[4] To account for inflation, data on firm sales are deflated using country-specific consumer price indices (CPIs).[5] Let S_t^{ijk} denote the annual sales in year t of firm k located in country i and belonging to industry j. The annual sales data are reported to the nearest \$1000 and, thus, are not binned. Normalized log sales, s_t^{ijk}, are defined as follows:

$$s_t^{ijk} = \log(S_t^{ijk}) - \frac{1}{|N_t^{ij}|}\sum_{i=1}^{|N_t^{ij}|}\log(S_t^{ijk}) \tag{1}$$

where N_t^{ij} denotes the set of firms in the sample determined by country i, industry j, and year t. The firm growth rate, r_t^{ijk}, is defined as follows:

$$r_t^{ijk} = s_t^{ijk} - s_{t-1}^{ijk}. \tag{2}$$

Defining the firm growth rate in this manner eliminates possible country-specific industrywide trends in the mean of annual sales (see Refs. [17,18]).

For each sample determined by particular values of i, j, and t, we test the hypothesis that the data $\{r_t^{ijk}\}_{k\in N_t^{ij}}$ are distributed according to a particular empirical distribution function F. In performing the EDF tests, we restrict our analysis to samples satisfying $|N_t^{ij}| \geq 10$. Imposing this restriction yields 63,568 observations of firm growth rates across 27 countries for a total of 9267 firms that are grouped into 2004 country/industry/year samples, with an average of approximately 32 firms per sample.

Summary statistics on the financial characteristics of these firms are presented in Table 1. Pooled across countries, industries, and years, the size distributions of firms, as measured by their capital or market value, are positively skewed. The mean firm capital is approximately six times larger than the median firm capital, and the mean firm market value is approximately seven times larger than the median firm market value.

3. EDF tests of theoretical distributions

To find the theoretical distribution that best approximates the empirical distributions of firm growth rates, we perform goodness-of-fit tests using EDF test statistics. Let F denote the empirical CDF and G denote the hypothesized distribution (e.g., the normal distribution). Let (X, δ) be any metric space of CDFs. Then, the distance $\delta(G, F)$ is an EDF test statistic. Commonly used EDF test statistics include the Anderson–Darling and Cramér–von Mises test statistics [19]. The Anderson–Darling test statistic is based on the parametric family $V_n = n\int_{-\infty}^{\infty}|G(x) - F(x)|^2\psi(x)\mathrm{d}G(x)$, where n is the number of observations. Setting $\psi(x) = \{F(x)(1 - F(x))\}^{-1}$ yields the Anderson–Darling test statistic. The Cramér–von Mises test statistic is derived by setting $\psi(x) = 1$ in the family $V_n = n\int_{-\infty}^{\infty}|G(x) - F(x)|^2\psi(x)\mathrm{d}G(x)$.

The parameters of each distribution are estimated using maximum-likelihood estimation [20]. Direct optimization of the log-likelihood is performed with the Nelder–Mead method. We first test the null hypothesis that the firm growth rates are normally distributed. As shown in Table 2, the null hypothesis is not rejected based on the Anderson–Darling

2 The Lévy α-stable family ($\alpha \in (0, 2]$) includes the Gaussian, or normal, ($\alpha = 2$) and Cauchy ($\alpha = 1$) distributions as special cases. All non-Gaussian Lévy α-stable distributions (i.e., $\alpha < 2$) present heavy tails [13].

3 To explain the consensus "tent-shape" finding, Bottazzi and Secchi [7] posit a mathematical framework based on the random assignment of business opportunities among firms. Manas [15] derives a mixed normal-Laplace distribution of firm growth rates using an alternative approach based on the direct stochastic modeling of firm sales. Metzig and Gordon [16] develop an agent-based model that yields a tent-shaped aggregate distribution of firm growth rates.

4 Source: evaDimensions, www.evadimensions.com.

5 CPIs are obtained from http://research.stlouisfed.org/.