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Volatility behavior of visibility graph EMD financial time series from Ising interacting system

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HIGHLIGHTS

- A financial price model is developed and investigated by stochastic Ising system.
- Empirical mode decomposition is used to decompose financial data into different frequency series.
- Volatility behavior of returns for financial time series is studied by visibility graph and horizontal visibility graph.
- Complexity analysis is performed for the considered series.
- Statistical analysis illustrates that the simulation data could exhibit the similar properties as the real data.

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ABSTRACT

A financial market dynamics model is developed and investigated by stochastic Ising system, where the Ising model is the most popular ferromagnetic model in statistical physics systems. Applying two graph based analysis and multiscale entropy method, we investigate and compare the statistical volatility behavior of return time series and the corresponding IMF series derived from the empirical mode decomposition (EMD) method. And the real stock market indices are considered to be comparatively studied with the simulation data of the proposed model. Further, we find that the degree distribution of visibility graph for the simulation series has the power law tails, and the assortative network exhibits the mixing pattern property. All these features are in agreement with the real market data, the research confirms that the financial model established by the Ising system is reasonable.

1. Introduction

Financial market is a complex evolving dynamic system, consisting of a large number of agents (investors) interacting with one another in complicated ways, which reacts to external investment information to determine the best price for a given asset. Nowadays, the analysis of financial systems based on stochastic dynamics has become one the of active-research topics in financial research. As the stock markets are becoming deregulated worldwide, the modeling of dynamics of forward prices is becoming a key problem in the risk management, physical assets valuation, derivatives pricing and so on, see Refs. [1–26]. Meanwhile, the opinion that the price fluctuations are due to the interactions among the market investors is being widely accepted. For the financial modeling, aiming to understand price fluctuations, in an attempt to reproduce the stochastic behaviors of financial time series, any model needs to define a mechanism for the formation of the price. And the

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behaviors of price changes in financial markets have long been a focus of economic research for a more clear understanding of mechanism and characteristics of financial markets. A complex behavior can emerge due to the interactions among smallest components of that system [27], and it is also a successful strategy to analyze the behavior of a complex system by studying these components. In the empirical research, some statistical properties for market fluctuations are revealed by high frequency financial time series, for instance, fat-tail phenomenon, power-law distribution and volatility clustering and so on [7,16,28–30]. Further, a volatility forecast and a second prediction on the volatility of volatility over the defined period is needed to price derivative contracts. Early models of volatility measurement include the work of Engle [31]. Engle developed a class of models known as autoregressive conditional heteroskedasticity (ARCH). Bollerslev [32] extended the ARCH to include past variance in the conditional variance function and this extension is widely known as the GARCH model. These models have gained popularity because they are capable of capturing volatility clustering and can accommodate fat-tailedness of return distribution at the same rate of persistence [33,34].

Recently, some researchers apply statistical physics systems (or interacting particle systems) to measure and explain this set of empirical facts. On one hand, economic systems such as financial markets are similar to physical systems in that they are comprised of a large number of interacting "agents". On the other hand, they are quite different and much more complex because economic agents are "thinking" units and they interact in complicated ways not yet quantified. Indeed, most physics approaches to finance view financial markets as a complex evolving system [35]. For example, Stauffer and Penna [11] applied the lattice percolation theory to the market fluctuations, and their model was based on the herding effect by which traders follow trends without considering any economic data. The existence of heavy tail behavior for the returns was found when the influence rate of the model is around or at a critical value. Krawiecki [10] and Fang and Wang [4,5] developed an interacting-agent model of speculative activity explaining price formation in financial market that is based on the stochastic Ising dynamic system. Lux and Marchesi [14] introduced an agent-based model in which chartist agents compete with fundamentalists agents, leading to power law distributed returns as observed in real markets which contradicts the popular efficient market hypothesis.

In the present work, we make an approach to establish a price model through the stochastic Ising system. In the proposed model, all of spins are flipped by following Ising dynamic system. We suppose that traders determine their positions at each time by observing and diffusing the market information, each trader is thought to be a subunit in the stock market, and may take positive (buying) position or negative (selling) position, denoted by $+$ and $-$ separately. We use Ising dynamic system to investigate the fluctuation of the stock market because that system consists of subunits. In this work, applying two kinds of graph based time series analysis methods, visibility graph [36,37] and horizontal visibility graph [38], we study the statistical behaviors of absolute return series and their corresponding IMFs (figured out by the empirical mode decomposition (EMD) method, which is used to decompose the data into a few lower frequency series [39,40]), and present some empirical relationship between the topological characteristic and the original time series. Furthermore, the complex properties of original return series and IMFs series after EMD are investigated by multiscale entropy analysis (MSE). It is a method to measure the disorder and complexity at multiple time scales [41,42]. To verify the feasibility of the proposed model, the simulation data and the real stock data of Shanghai Composite Index (SSE) and Hang Seng Index (HSI) are compared.

2. Financial time series model derived from Ising system

2.1. Brief description of Ising system

Ising dynamic system is one of stochastic interacting particle systems [43–48]. Let $\mathbb{Z}^2$ be the usual 2D square lattice and we denote by $\mathfrak{B}$ the set of bonds of the lattice (pairs of nearest neighbors). Let $\Omega_{\mathbb{Z}^2} = \{+1, -1\}^{\mathbb{Z}^2}$ denote the space of spin configurations on $\mathbb{Z}^2$, an element of $\Omega_{\mathbb{Z}^2}$ usually denote by $\sigma = \{\sigma_i : i \in \mathbb{Z}^2\}$. The spin σ_i take one of the integer values $+1$ or -1. We consider the Ising model with the following dynamic system of Hamiltonian, for every $\sigma \in \Omega$

$$H_{\mathbb{Z}^2,b}(\sigma) = -J \sum_{\langle i,j \rangle} \delta_{\sigma_i,\sigma_j} - b \sum_i \delta_{\sigma_{i,1}} \tag{1}$$

where $J > 0$ for the ferromagnetic systems and δ is the Kroeneker symbol. The first sum is over all nearest neighbors (denote by $\langle i, j \rangle$) on the lattice and the second sum over all lattice points. Here the applied magnetic field b acts on the (arbitrarily chosen) state 1. Then the partition function is

$$Z_{\mathbb{Z}^2,h}(\sigma) = \sum_{\sigma} \exp\left(K \sum_{\langle i,j \rangle} \delta_{\sigma_i,\sigma_j} + h \sum_i \delta_{\sigma_{i,1}} \right) \tag{2}$$

where $K = \beta J$ and $h = \beta b$, $\beta = 1/(k_B T)$, k_B and T being Boltzmann's constant and temperature, respectively.

The spins in Ising model can point up (spin value $+1$) or point down (spin value -1), and the grain flips between the two orientations. As β approaches the critical inverse temperature β_c from below, spin fluctuations are present at all scales of length. When $\beta = \beta_c$, the correlations decay by a power law, while for $\beta > \beta_c$, there exist two distinct pure phases, i.e., the Ising model exhibits the phase transition, for more details see Refs. [43–45]. Correlations play a significant role in studying the fluctuations of the phase interfaces for the statistical physics model, see Refs. [44,48]. In the following of present work,