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# Quark-gluon plasma phase transition using cluster expansion method

A.M. Syam Kumar, J.P. Prasanth *, Vishnu M. Bannur

*Department of Physics, University of Calicut, 673635, India*

## HIGHLIGHTS

- Condensation of QGP is presented.
- EoS using modified Cornell potential is investigated.
- Ushcats convergence method for cluster expansion is used.
- Isotherm for different temperature and different number of charm quarks is obtained.
- A comparison of Mayer's relation with that of Ushcats is made.

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## ABSTRACT

This study investigates the phase transitions in QCD using Mayer's cluster expansion method. The inter quark potential is modified Cornell potential. The equation of state (EoS) is evaluated for a homogeneous system. The behaviour is studied by varying the temperature as well as the number of Charm Quarks. The results clearly show signs of phase transition from Hadrons to Quark-Gluon Plasma (QGP).



## 1. Introduction

The study of the fundamental theory of strong interactions, Quantum Chromodynamics (QCD), under extreme conditions of temperature and density has been one of the most challenging problems in physics. A primary goal of relativistic nuclear collisions is the observation of a phase transition of confined, hadronic matter to a deconfined quark–gluon plasma [1,2]. The phase transition has much importance in statistical mechanics as well as other fields including Quantum Field Theory at finite temperature.

The most important rigorous approach to derive the Virial equation of state (VEOS) was given by Mayer [3,4] and his collaborators. Using cluster expansion method, Mayer was able express the partition function as a series of expansion in powers of density. The first limitation of this method is that the density should be small so as to satisfy the convergence condition. In addition to this there exists a strong limitation that restricts density more strictly given by

$$\sum_{k\geq 1} k\beta_k \rho^k < 1. \qquad (1)$$

* Corresponding author.
*E-mail address:* prasanthjp36@gmail.com (J.P. Prasanth).

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This limitation on density is related to the radius of convergence and makes it impossible to predict the region of condensation. So the phenomena of phase transition cannot be proved using classical statistical mechanics. Ushcats, using analytical complex calculation, [5,6] shows that it is possible to neglect the forbidden region in Mayer's equation of state where the equation of state cannot be defined. The equation as derived by Ushcats clearly demonstrates the process of condensation at high density region. A modified generating function for the canonical partition function has been derived in Ref. [7]. Using this generating function, analyse again Mayer's theory of classical cluster expansion and study Mayer's theory of virial expansion and condensation at thermodynamic limit.

The main purpose of this work is to produce an equation of state for quark–gluon plasma, using the mathematical formalism of Ushcats [6]. Here we use the interacting potential as modified Cornell potential [8], in which the Cornell potential [9,10] is modified by including a screening effect. The volume is varied keeping temperature fixed and the corresponding pressure is calculated. The same can be done by changing the density $\rho$, as done by Ushcats [5,6] using the Lennard-Jones potential.

## 2. Theory of condensation

Mayer expressed the partition function $Q_N$ as an integral over the volume of a two particle function $f_{ij}$ [3,4]

$$Q_N = \frac{1}{N!\lambda^{3N}} \int_{V^N} \prod_{i<j} (1 + f_{ij}) \mathrm{d}\mathbf{r}^N \tag{2}$$

where $f_{ij}$ is defined by the relation

$$f_{ij} = \exp\left(\frac{-U_{ij}}{k_B T}\right) - 1. \tag{3}$$

Using $N$-particle graph technique and introducing *cluster integral* $b_l$, in the thermodynamic limit ($N \to \infty$) the partition function is given by

$$Q_N(V,T) = \sum_{\{m_l\}}' \left[\prod_{l=1}^{N} \left\{\left(b_l \frac{V}{\lambda^3}\right)^{m_l}\right\} \frac{1}{m_l!}\right] \tag{4}$$

where the restricted summation goes over all the sets $\{m_l\}$ with the condition

$$\sum_l l m_l = N. \tag{5}$$

The cluster integral $b_l$ can be expressed as a sum of terms with irreducible cluster integral $\beta_k$'s.

$$b_l = \frac{1}{l^2} \sum_{\{n_k\}}' \prod_k \frac{(l\beta_k)^{n_k}}{n_k!} \tag{6}$$

where the restricted summation goes over all the sets $\{n_k\}$ with the condition

$$\sum_k k n_k = l - 1. \tag{7}$$

Using complex combinatorial methods Mayer derived the Virial expansion for pressure. The virial coefficient $B_{k+1}$'s are related to the corresponding *irreducible cluster integral* $\beta_k$'s.

$$B_{k+1} = -\frac{k}{k+1}\beta_k. \tag{8}$$

From these definitions, the maximum term in $Q_N$ is found and equating to logarithm of density and activity results in

$$\ln\left(\frac{Q_N}{N!}\right) = N\left[1 + \sum_{k\geq 1} \frac{1}{k+1}\beta_k \rho^k + \ln(\rho)\right]. \tag{9}$$

From the above relation the thermodynamic properties of the imperfect gas can be derived. The equation of state (EoS) obtained from the above relation has the form

$$\frac{PV}{Nk_BT} = 1 - \sum_{k\geq 1} \frac{k}{k+1}\beta_k \rho^k. \tag{10}$$