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Network topology control strategy based on spatial evolutionary public goods game

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HIGHLIGHTS

- The phenomenon that competitive individual will attract more partners is interpreted with the framework of spatial evolutionary game theory.
- A strategy that impels nodes of network to connect with a specified node is proposed.
- Cooperative environment is essential for attractive operation.
- Low rewire frequency leads to high level of heterogeneity.

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ABSTRACT

It is often the case that rational individuals will adjust their connectivity in commercial or social activities for maximizing their payoffs. In this process, we can observe that individuals always gather around a leader or a competitive individual who can bring them more benefits. Inspired by this, we propose a strategy that impels nodes of network to connect with a specific node that we have specified with the perspective of spatial evolutionary public goods game. Thus a node is specified and given a larger enhancement factor which reflects his advantage over others. Then we employ a payoff-oriented preferential rewire strategy that individual will sever a neighbor who provides him with the lowest benefit and then link others randomly. The results illustrate that this strategy not only ensures the promotion of cooperation but also increases the degree of the specified node. Furthermore, we analyze the effect of two relevant parameters: enhancement factor and rewire frequency. We find that if the control strategy expects to work effectively, these two parameters have to ensure an evolution environment where cooperators can prevail defectors. We also conclude that a relatively low rewire frequency contributes to increasing the degree of the specified node. Meanwhile we attempt to present our interpretations for these phenomena.

1. Introduction

Game theory has been recognized as a powerful tool for analyzing economic problems and human behaviors [1,2]; Public goods game (PGG) is generally representative for N-players interaction game. Cooperators (C) contribute an amount c to the public good; defectors (D) do not contribute. The total contribution is multiplied by enhancement factor r ($r > 1$) and the result is equally distributed among all N members of the group irrespective of their contributions [3,4]. Thus defectors

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have a fitness advantage over cooperators and with restriction of $r < N$, cooperation behaviors will vanish under natural selection that prefers more competing strategies [5]. Hence, defective behaviors emerge as the rational option and there will be no resources to share. This phenomenon is inconsistent with the fact that cooperative interactions thrive at all levels in real living systems [6–8]. Spatial evolutionary game theory posit this problem of cooperation by embedding it in a structured population and networked interactions [9,10]. In reality, different individuals interact with different subsets of the whole population. Such topology constrains can be described by means of complex network [11]. Therefore complex network can be seen as a spatially extended systems where individuals, assigned to the nodes, do not interact randomly and completely, as in a well-mixed population [12], but interact with individuals who are associated with links and participate in the public goods games of which his neighbors consist. In recent years, a number of evolutionary game theory models have demonstrated that the structured population can promote the evolution of cooperation in group interactions [3,10,13–15].

The principle that spatial evolutionary game theory can promote cooperation is whether spatial arrangement of individuals can enable cooperators to form clusters and thereby to minimize exploitation by defectors [12]. As an extension of evolutionary games on static networks, dynamics of network topology which changes in response to the state of individuals and individual strategy has attracted mounting attention [4,16–22]. Attila Szolnoki and Matjaž Perc have found that both additions of new links as well as deletions of existing links can favor cooperative behavior [23]. Several rewire mechanisms, which equip individuals with the capability of self-organizing their social ties, have been proposed to pursue cooperative outcomes [8–10,24–26]. Attila Szolnoki and Matjaž Perc examined a version of coevolutionary rule that qualitatively preserving the initial heterogeneity of the interaction network can promote cooperation [27]. Rand et al. have confirmed that cooperation is maintained at a high level when individuals preferentially break links with defectors and form new links with cooperators [10].

In related coevolution works, it is observed that, along with coevolution dynamics, the emerging social networks exhibit an overall heterogeneity [8]. Lei Wang and Chengyi Xia found that, under the framework of PGG, networks generated by an evolutionary preferential attachment mechanism exhibit the transition from homogeneous networks to heterogeneous ones [28]. This heterogeneity tendency fits nicely to the studies that social diversity may lead to much fairer outcomes [3,29,30]. However, in this work, we not only focus on the promotion of cooperation and the overall heterogeneity transition, but also consider how we can make network topology evolve in an expected way. With the assumption that every node of network is treated as a rational individual that is always bent on maximizing his payoffs under the rule of evolutionary game theory, individuals have the initiative desire to adjust their connectivity with the purpose of obtaining more benefits. Thus, a leader with outstanding charisma, preferential policy, advanced instrument, etc., who can guide or cooperate with his partners to enhance their assets more effectively and significantly, will be favored. Naturally, this competitive individual will attract more followers and adherents, then become the center of the population. Applying this evolution rule, we propose a notion that we can direct individuals to follow a certain individual whom we want to elect as a leader. Constructing this process in spatial evolutionary public goods game, we present a strategy that can impel nodes of network to become neighbors of a specific node that we have picked up particularly as well as ensure the promotion of cooperation.

2. Model

Initial population structure is represented by ER-random graph. N individuals which occupy the vertices of the graph and interactions proceed along the edges [3]. Half among these N individuals are assigned to be cooperators randomly and the rest defectors.

Assuming k_x is the degree of node x, cooperator contributes $c = 1$ to all his $k_x + 1$ groups of PGG, constituted by the focal individual x and his k_x neighbors, while defector nothing. The sum of all these contributions is multiplied by an enhancement factor r, reflecting the synergetic effects of cooperation [31], and then shared equally among all these players irrespective of their contributions. Thus the payoff p_{xy} of individual x obtaining from the PGG which is centered on x's neighbor individual y is given by [32]

$$p_{xy} = \begin{cases} c\eta n_{y,c} - c & \text{if } x \text{ is a } C \\ c\eta n_{y,c} & \text{if } x \text{ is a } D \end{cases} \quad (1)$$

where $n_{y,c}$ is the number of cooperators in the group composed of the neighborhood of individual y plus himself. $\eta = \frac{r}{k_y+1}$ is the renormalized PGG enhancement factor. The payoffs P_x of individual x is the sum of gains acquired from all of the PGGs that he participated in during each time step [32],

$$P_x = \sum_{y \in \Omega_x} p_{xy} \quad (2)$$

where Ω_x denotes the player x's neighbors plus himself.

Each individual updates strategy through imitating the behavior of their neighbors. For example, individual x can pick up a role individual y at random in his k_x neighbors and on the basis of the payoff-oriented preferential learning mechanism, individual x will switch his strategy s_x to the strategy s_y of individual y with probability

$$W(s_x \to s_y) = \frac{1}{1 + \exp[\beta(P_x - P_y)]} \quad (3)$$