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Effects of inspections in small world social networks with different contagion rules

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HIGHLIGHTS

- Inspections are used to prevent tax evasion or any other unlawful behavior.
- The effect of inspections depends on the network topology and the contagion rule.
- The network is modeled as a Watts–Strogatz Small World that is tuned from regular to random.
- Two contagion rules are applied: continuous and discontinuous.
- The equilibrium populations of payers and evaders are obtained in terms of these system parameters.

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ABSTRACT

We study the way the structure of social links determines the effects of random inspections on a population formed by two types of individuals, e.g. tax-payers and tax-evaders (free riders). It is assumed that inspections occur in a larger scale than the population relaxation time and, therefore, a unique initial inspection is performed on a population that is completely formed by tax-evaders. Besides, the inspected tax-evaders become tax-payers forever. The social network is modeled as a Watts–Strogatz Small World whose topology can be tuned in terms of a parameter $p \in [0, 1]$ from regular ($p = 0$) to random ($p = 1$). Two local contagion rules are considered: (i) a continuous one that takes the proportion of neighbors to determine the next status of an individual (node) and (ii) a discontinuous (threshold rule) that assumes a minimum number of neighbors to modify the current state. In the former case, irrespective of the inspection intensity ν, the equilibrium population is always formed by tax-payers. In the mean field approach, we obtain the characteristic time of convergence as a function of ν and p. For the threshold contagion rule, we show that the response of the population to the intensity of inspections ν is a function of the structure of the social network p and the willingness of the individuals to change their state, r. It is shown that sharp transitions occur at critical values of ν that depends on p and r. We discuss these results within the context of tax evasion and fraud where the strategies of inspection could be of major relevance.

1. Introduction

In the past few years mathematical and computer modeling have been used to provide new and relevant insights in the mechanisms of diffusion of fraud and tax evasion in modern societies [1–5] thus, suggesting how to optimize strategies to

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contrast such phenomena. A particular attention has been focused on how unlawful behavior can induce a sort of contagion on the "neighboring" individuals in a given society. Indeed, the personal attitude facing social compliance depends very much on the opinion of other partners (citizens) and, in a global scale, on public opinion [6]. The way our neighbors, e.g. friends, colleagues or, in general, peers, face the social contract highly affects our position to this respect. Indeed, the stability of human groups depends on the principle of cooperation that is implemented, among other factors, in tax compliance [7]. When cooperation is not integral, i.e. when some participants do not contribute to the common pool then, group coexistence becomes fragile.

Interventions are widely applied to control network behavior [8]. In particular, inspections and, if necessary, punishment is an effective way of fighting against tax evasion. Inspections are costly and, therefore, they have to be implemented in a rational way [9]. A complete inspection, i.e. to every citizen of the society, becomes unpractical even for small size populations. Instead, random inspections are feasible in general. The question arises about which is the optimal intensity, i.e. number of inspected individuals, to be applied. Moreover, the timing of inspections could become relevant in certain situations. These decisions are obviously taken under a limited budget scenario that complicates even more the optimal solution of the problem. In this paper we assume that a unique set of inspections is initially performed to the whole population. The intensity of the inspections, i.e. the size of the set of individuals that is inspected, is taken as a control parameter of the problem. Consequently, we study the equilibrium distribution of the population. This is consistent with assuming that inspections occur at a larger time scale than the relaxation period of the population. For simplicity, we consider that individuals can only take two states: law-abiding (tax payers) and free-riders (tax-evaders). Human societies are linked, i.e. individuals are not isolated but they relate to each other by social links that transmit, for instance, their propensity to pay taxes. The way the social contagion occurs and the structure of the social network are two main aspects that determine the final outcome of inspections. An individual can belong to different social networks, e.g. professional or private, and the structure of these networks can be very different [10]. For instance, the social network of scientists is proven to have a scale free topology and the same topology appears in the network formed by sexual contacts among individuals (see, for instance, [11] for a review). However, friendship and peer networks seem to exhibit a small world structure [12]. Information concerning tax evasion is a delicate matter and consequently, it tends to flow through networks of small world type. Therefore, we will focus on them in this work.

A widely applied assumption treats social contagion from an ecological perspective as an epidemic, where the probability of an individual (i) to be infected at time t, $H(i;t)$, is proportional to the number of its infected neighbors at that time, $N(i;t)_I$:

$$H(i;t) = \lambda N(i;t)_I$$

where $\lambda > 0$ is a contagion rate. However, recent investigations have shown that the micro-processes involved in social contagion are complex [13–16]. A family of models take into account social peculiarities by assuming a discontinuous response of a node to the inflow information [17]. These models are inspired in the classical threshold model [18] where the adoption of an initiative depends basically on whether a minimum number of neighbors have already adopted it. Recent works have shown that social diffusion in real networks are of this type [19–21]. Nonetheless, in other cases, probabilistic diffusion mechanisms seem to describe better the behavior of the network [22]. Besides, the individuals personality is known to play an important role in the adoption of certain features [23]. For this reason, in our model we assign a personality to the nodes, measured by a parameter r, that drives the willingness to change their state.

The remainder of the paper is organized as follows: Next section presents in detail a simple model and defines its main parameters. Sections 3 and 4 study the effects of inspections on the population for different network topology for two different contagion rules. We discuss the implications of our results in the last section.

2. A toy model

Let us assume a population of interacting individuals that form a connected social network. We assume that the network is homogeneous that is, all the nodes have approximately the same number of neighbors (equal average connectivity). In other words, the network has a characteristic scale with regard to its connectivity. In order to tune the degree of randomness we take the Small-World model of Watts and Strogatz (WS) [24] as the reference network. This family of networks has been identified with real social networks because its short path length (the diameter of the network increases logarithmically with the number of nodes) and the high clustering coefficient. A regular ring of N nodes symmetrically connected with k nearest neighbors is transformed by rewiring the links to randomly chosen nodes with a probability p. By construction, the WS network lies in between two limit types of networks: regular (ring) and random [24]. If $p = 0$ the regular network is preserved, whereas for $p = 1$ a random network is formed. WS networks have connectivity distributions $P(k)$ that are peaked at an average value $\hat{k} = \langle k_i \rangle$ and decaying exponentially for $k > \hat{k}$ and $k < \hat{k}$ (Poissonian like distribution).

The influence of inspections in the time evolution of the population depends on (i) the intensity applied (ν), (ii) the network topology (p) and (iii) the local diffusion (updating) rules (e.g. continuous or discontinuous). The inspections reduce the number of nodes that can become evaders at any time. In all cases, for each intensity of the inspections, the population evolves towards an equilibrium state characterized by a distribution of evaders and payers. The time it takes the population to converge depends on the parameters. For the parameter setup used in the simulations, it is observed that after 10 000 time steps the stationarity of the population is assured. The final estimation of the equilibrium population is obtained from