Physica A 432 (2015) 391–399

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

One-norm geometric quantum discord of two-qubit state in spin chain environment at finite temperature

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HIGHLIGHTS

- One-norm GQD is more robust than QD against the decoherence at finite temperature.
- One-norm GQD is superior to QD in detecting the critical point of QPT of the spin chain.
- The phenomena of sudden transition and double sudden transitions are analyzed.
- The methods to control the frozen time of quantum correlations are proposed.

ARTICLE INFO

Article history:
Received 5 January 2015
Received in revised form 13 February 2015
Available online 27 March 2015

Keywords:
Quantum correlations
Quantum discord
Decoherence
Quantum phase transition
Sudden transition
Spin chain

ABSTRACT

We study the dynamics of quantum correlations measured by quantum discord (QD) and one-norm geometric quantum discord (GQD) in a two-qubit system coupled to an XY spin chain with finite temperature. It is shown that one-norm GQD is more robust than QD against the decoherence induced by the spin chain with finite temperature, and one-norm GQD is superior to QD in detecting the critical point of quantum phase transition (QPT) of the spin chain. Considering the effects of the state parameters, the temperature of the spin chain and the asymmetric coupling parameter, we analyze the phenomenons of sudden transition and double sudden transitions occurred in the evolutions of QD and one-norm GQD. Besides, the influences of the Dzyaloshinsky–Moriya (DM) interaction and the three-site interaction in the spin chain are also taken into account.

1. Introduction

With the rapid development of quantum information theory, quantum entanglement as a type of quantum correlations has gained significant study and exploitation of quantum advantage [1]. However, quantum correlations may have more general and fundamental definition instead of using entanglement, since there exist quantum tasks that display the quantum advantage without entanglement [2]. It has been found that quantum discord (QD) defined as the difference between quantum mutual information and classical correlation [3] is supposed to characterize all of nonclassical correlations in a bipartite state. A nonzero QD but not entanglement may be responsible for the efficiency of a quantum computer [4,5]. Therefore, much attention has been paid to the study of QD [6–10]. However, due to the complicated optimization involved, it is usually complicated to calculate QD analytically. In order to overcome this difficulty, Dakić et al. [11] introduce a geometric quantum discord (GQD) which is defined as the minimal distance between a given state and all states with zero discord. It is

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http://dx.doi.org/10.1016/j.physa.2015.03.028
0378-4371/

proved that GQD is analytically computable for arbitrary bipartite states [12]. But recent study shows that GQD as proposed in Ref. [11] may increase under local operations on the unmeasured subsystem [13], so it cannot be regarded as a good measure for quantum correlations. In this context, the one-norm GQD is proposed since it does not suffer from an inherent problem which affects the GQD originally introduced in Ref. [11]. It is reported that the one-norm GQD as a measurement of QD has exhibited some interesting phenomenons, such as double sudden transitions in the decoherence problems [14–17].

On the other hand, real quantum systems will unavoidably interact with the surrounding environment and thus lead to decoherence. In the past few years, decoherence or disentanglement induced by the spin environment with quantum phase transition (QPT) has been discussed [18–27]. It is found that at the critical point where the environment occurs QPT, the decoherence is enhanced and the disentanglement process is accelerated by the quantum criticality. Recently, there has been an increase in the investigations of the dynamic behavior of QD under spin environment [28,29]. The results show that the QD is more robust than entanglement and exhibits sudden change. In this paper, as an extension of the work [29], we present a theoretical investigation of quantum correlations dynamics of two-qubit system coupled to an XY spin chain by one-norm GQD. Comparing with the case of QD, we find the one norm-GQD can reveal more properties about quantum correlations of the system under finite temperature environment.

2. Hamiltonian evolution

The total Hamiltonian of the composite system we considered in this paper is described by [29]

$$H = H_E + H_I, \tag{1}$$

where

$$H_E = -\sum_{l=1}^{N}\left(\frac{1+\gamma}{2}\sigma_l^x\sigma_{l+1}^x + \frac{1-\gamma}{2}\sigma_l^y\sigma_{l+1}^y + \lambda\sigma_l^z\right) \tag{2}$$

denotes the Hamiltonian of the environmental spin chain, and

$$H_I = -g(\sigma_A^z + f\sigma_B^z)\sum_{l=1}^{N}\sigma_l^z \tag{3}$$

is the interaction Hamiltonian between the two-qubit system and the spin chain. Here $\sigma_{A,B}^z$ and $\sigma_l^{x,y,z}$ are the Pauli operators used to describe the two qubits and the spin chain respectively. N is the number of spins in the chain, and the periodic boundary conditions $\sigma_{N+1}^{x,y,z} = \sigma_1^{x,y,z}$ are satisfied. γ measures the anisotropy in the in-plane interaction, and λ represents the strength of the transverse field applied to environment. g is coupling strength between the two-qubit system and the spin chain. The parameter $f \in [0, 1]$ denotes the two qubits couple asymmetrically to the spin chain. Notice that $[g(\sigma_A^z + f\sigma_B^z), \sigma_l^{x,y,z}] = 0$, so the total Hamiltonian can be rewritten as

$$H = \sum_{\mu=1}^{4} |\phi_\mu\rangle\langle\phi_\mu| \otimes H_E^{\lambda_\mu}, \tag{4}$$

where $|\phi_\mu\rangle$ is the μth eigenstate of the operator $g(\sigma_A^z + f\sigma_B^z)$ and reads $|ee\rangle$, $|eg\rangle$, $|ge\rangle$ or $|gg\rangle$ corresponding to the μth eigenvalue g_μ, and λ_μ is given by $\lambda_\mu = \lambda + g_\mu$ taking the following expressions $\lambda_{1,4} = \lambda \pm (1+f)g$, $\lambda_{2,3} = \lambda \pm (1-f)g$. $H_E^{\lambda_\mu}$ is given from H_E by replacing λ with λ_μ. By successively using the Jordan–Wigner transformation $\sigma_l^z = 1 - 2c_l^+ c_l$, $\sigma_l^+ = \prod_{m<l}(1 - 2c_m^+ c_m)c_l$, $\sigma_l^- = \prod_{m<l}(1 - 2c_m^+ c_m)c_l^\dagger$, Fourier transformation $d_k = \frac{1}{\sqrt{N}}\sum_l c_l \exp(-i2\pi lk/N)$ with $k = -M, \ldots, M$ and $M = (N-1)/2$ for odd N, and the Bogoliubov transformation $b_{k,\lambda_\mu} = d_k \cos\frac{\theta_k^{\lambda_\mu}}{2} - id_{-k}^\dagger \sin\frac{\theta_k^{\lambda_\mu}}{2}$ with $\tan(\theta_k^{\lambda_\mu}) = (\gamma \sin\frac{2k\pi}{N})/(\lambda_\mu - \cos\frac{2k\pi}{N})$, the Hamiltonian $H_E^{\lambda_\mu}$ can be diagonalized exactly as [22]

$$H_E^{\lambda_\mu} = \sum_{k=-M}^{M} \xi_k^{\lambda_\mu}\left(b_{k,\lambda_\mu}^\dagger b_{k,\lambda_\mu} - \frac{1}{2}\right), \tag{5}$$

with the energy spectrum is

$$\xi_k^{\lambda_\mu} = 2\sqrt{\left(\lambda_\mu - \cos\frac{2\pi k}{N}\right)^2 + \gamma^2\sin^2\frac{2\pi k}{N}}. \tag{6}$$

Now we can calculate the dynamic evolution of the two-qubit system. Let us assume that the two qubits and the environmental spin chain are initially in the product density matrix form

$$\rho(0) = \rho_{AB}(0) \otimes \rho_E(0), \tag{7}$$